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Algebra Sequences and Series

POLYNOMIALS

SPEC '10 1 (b) Use the remainder theorem, or otherwise, to find the reminder when $x^3 - 2x^2 + 4x - 21$ is divided by $x - 3$.

[2 marks]

2012 1 (b) If $x + 2$ is a factor of $f(x) = 2x^3 - 3x^2 - 4x + a$, find the value of a.

(2 marks)

2013 1 (a) Let $f(x) = x^3 - x^2 - 14x + 24$.

    (i)   Use the factor theorem to show that $x + 4$ is a factor of $f(x)$.   (2 marks)

    (ii)   Determine the other linear factors of $f(x)$.   (3 marks)

2014 1 (b) When the expression $2x^3 + ax^2 - 5x - 2$ is divided by $2x - 1$, the remainder is -3.5.

Determine the value of the constant a.   (3 marks)

2015 1 (c) Given that $x - 2$ is a factor of $k(x) = 2x^3 - 5x^2 + x + 2$, factorize $k(x)$ completely.

(3 marks)

2017 1 (b) Given that the remainder when $f(x) = x^3 - x^2 - ax + b$ is divided by $x + 1$ is 6, and that $x - 2$ is a factor, determine the values of a and b.

(4 marks)

2018 1 (b) Derive the polynomial, $P(x)$, of degrees 3 which has roots equal to 1, 2 and -4.

(3 marks)

QUADRATICS

SPEC 2(a)
'10

Let $f(x) = 3x^2 + 12x - 18$.

    (i)   Express f(x) in the form $a(x + b)^2 + c$.   **[3 marks]**

    (ii)   State the minimum value of f(x).   **[1 mark]**

    (iii)   Determine the value of x for which f(x) is a minimum.   **[1 mark]**

SPEC (b)
'10

Find the set of values of x for which $2x^2 + 2 > 5x$.   **[4 marks]**

2012 2(a)

The quadratic equation $x^2 - 4x + 6 = 0$ has roots α and β.

Calculate the value of $\alpha^2 + \beta^2$.   **(5 marks)**

2012 2(b)

Find the range of values of x for which

$$\frac{2x - 5}{3x + 1} > 0.$$ **(4 marks)**

2013 2. (a)

Let $f(x) = 3x^2 + 6x - 1$.

    (i)   Express $f(x)$ in the form $a(x + h)^2 + k$ where h and k are constants.   **(3 marks)**

    (ii)   State the minimum value of $f(x)$.   **(1 mark)**

    (iii)   Determine the value of x for which $f(x)$ is a minimum.   **(1 mark)**

2013 2. (b)

Find the set of values of x for which $2x^2 + 3x - 5 \geq 0$.   **(4 marks)**

2014 2. (a)

Given that $f(x) = -2x^2 - 12x - 9$.

    (i)   Express $f(x)$ in the form $k + a(x + h)^2$, where a, h and k are integers to be determined.   **(3 marks)**

    (ii)   State the maximum value of $f(x)$.   **(1 mark)**

    (iii)   Determine the value of x for which $f(x)$ is a maximum.   **(1 mark)**

2014 2. (b)

Find the set of values of x for which $3 + 5x - 2x^2 \leq 0$.   **(4 marks)**

2015 2.(a) Given that $f(x) = 3x^2 - 9x + 4$:

    (i)   Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are real numbers.

          (3 marks)

    (ii)   State the coordinates of the minimum point of $f(x)$.   **(1 mark)**

2015 2.(b) The equation $3x^2 - 6x - 4 = 0$ has roots α and β. Find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$. **(4 marks)**

2015 2.(c) Determine the coordinates of the points of intersection of the curve

        $2x^2 - y + 19 = 0$ and the line $y + 11x = 4$.   **(3 marks)**

2016 2.(a)   (i)   Determine the nature of the roots of the quadratic equation $2x^2 + 3x - 9 = 0$.

          (1 mark)

    (ii)   Given that $f(x) = 2x^2 + 3x - 9$, sketch the graph of the quadratic function, clearly indicating the minimum value.

          (5 marks)

2017 2.(a) The quadratic equation $2x^2 + 6x + 7 = 0$ has roots α and β.

    Calculate the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$.

          (4 marks)

2017 2.(b) Determine the range of values of x for which $\dfrac{2x+3}{x+1} \geq 0$.

          (4 marks)

2019. 2.(a)   (i)   Express $f(x) = -2x^2 - 7x - 6$ in the form $a(x+h)^2 + k$.   **(3 marks)**

    (ii)   State the maximum value of $f(x)$.   **(1 mark)**

    (iii)   State the value of x for which $f(x)$ is a maximum.   **(1 mark)**

    (iv)   Use your answer in (a) (i) to determine all values of x when $f(x) = 0$.   **(3 marks)**

    (v)   Sketch the function $f(x)$ and show your solution set to (a) (iv) when $f(x) < 0$.

          (2 marks)

2018. 2. (a)　　(i)　　Given that $g(x) = -x^2 + x - 3$, express $g(x)$ in the form $a(x+h)^2 + k$ where a, h and k are constants.

(3 marks)

(ii)　　On the grid provided below, sketch the graph of $g(x)$, showing the maximum point and the y-intercept.
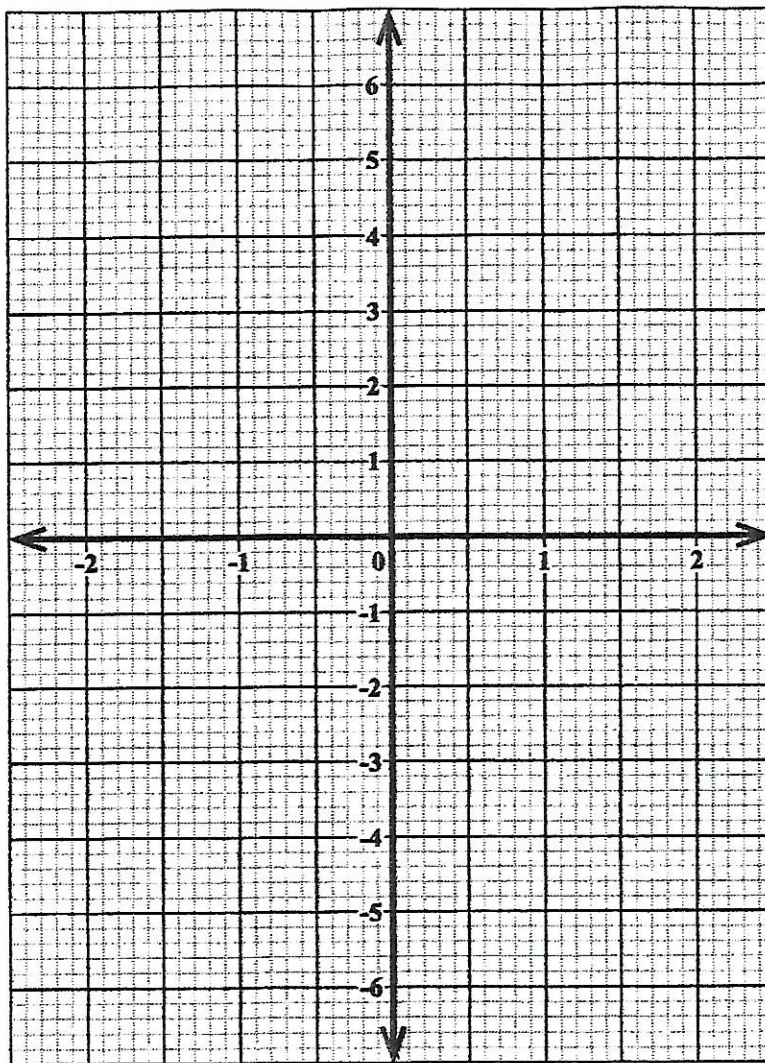
(3 marks)

2018. 2 (c)　　If α and β are the roots of the equation $2x^2 - 5x + 3 = 0$, determine the value of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$.

(4 marks)

AP & GP

SPEC '10 **2.(c)**   Given the series $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + ...$

    (i)   show that this series is geometric,   **[3 marks]**

    (ii)   hence, find the sum to infinity of this series.   **[2 marks]**

2012 2.(c)   A customer repays a loan monthly by increasing the payment each month by $x. If the customer repaid $50 in the 5th month and $70 in the 9th month, calculate the TOTAL amount of money repaid at the end of the 24th month.   **(5 marks)**

2013 2.(c)   Find the sum to infinity of the following series:

$$\frac{1}{4} + \frac{2}{4^2} + \frac{1}{4^3} + \frac{2}{4^4} + ...$$

    <u>Note</u>: This series can be rewritten as the sum of two geometric series.   **(5 marks)**

2014 2.(c)   A series is given by $0.2 + 0.02 + 0.002 + 0.0002 + ...$

    (i)   Show that this series is geometric.   **(3 marks)**

    (ii)   Find the sum to infinity of this series, giving your answer as an exact fraction.   **(2 marks)**

2015 2.(d)   An employee of a company is offered an annual starting salary of $36 000 which increases by $2 400 per annum. Determine the annual salary that the employee should receive in the ninth year.   **(3 marks)**

2016 2.(b)   Evaluate $\sum\limits_{1}^{25} 3^{-n}$.

                                    (3 marks)

    (c)   A man invested $x in a company in January 2010, on which he earns quarterly dividends. At the end of the second, third and fourth quarter in 2011, he earned $100, $115 and $130 respectively. Calculate the total dividends on his investment by the end of 2016.

                                    (5 marks)

2017 2(c) An accountant is offered a five-year contract with an annual increase. The accountant earned a salary of $53 982.80 and $60 598.89 in the third and fifth years respectively. If the increase follows a geometric series, calculate

(i) the amount paid in the first year

(4 marks)

(ii) the TOTAL salary earned at the end of the contract.

(2 marks)

2018 2.(b) In a geometric progression, the 3rd term is 25 and the sum of the 1st and 2nd terms is 150. Determine the sum of the first four terms, given that $r > 0$.

(4 marks)

2019 2.(b) A geometric series can be represented by $\dfrac{y}{x} + \dfrac{y^2}{x^3} + \dfrac{y^3}{x^5} + \ldots$

Prove that $S_\infty = xy\,(x^2 - y)^{-1}$.

(4 marks)

INDICES, SURDS AND LOGS

SPEC 1 (c) **'10**

A student collects laboratory data for two quantities q and p as shown in Table 1.

Table 1

q	1	2	3	4
p	0.50	0.63	0.72	0.80

The student reasons a relationship of the form $p = aq^n$

(i) Use logarithms to reduce this relation to a linear form.

[2 marks]

(ii) Using the graph paper provided and a scale of 1 cm to represent 0.1 units on the horizontal axis and a scale of 2 cm to represent 0.1 units on the vertical axis, plot a suitable straight line graph and hence estimate the constants a and n.

[6 marks]

2012 1 (c) Solve the equation $3^{2x} - 9(3^{-2x}) = 8$. (5 marks)

(d) (i) Express $x^3 = 10^{x-3}$ in the form $\log_{10} x = ax + b$. (2 marks)

(ii) Hence, state the value of the gradient of a graph of $\log_{10} x$ versus x. (1 mark)

2013 1 (c) Given that $5^{3x-2} = 7^{x+2}$, show that

$$x = \frac{2(\log 5 + \log 7)}{(\log 125 - \log 7)}.$$ (4 marks)

2014 1. (c) The length of a regtangular kitchen is y m and the width is x m. If the length of the kitchen is half the square of its width and its perimeter is 48 m, find the values of x and y (the dimensions of the kitchen). (5 marks)

2015 1. (d) Solve the following equations:

(i) $16^{x+2} = \frac{1}{4}$ (2 marks)

(ii) $\log_3 (x + 2) + \log_3 (x - 1) = \log_3 (6x - 8)$ (4 marks)

2019 1. (c) Solve $3x \log 2 + \log 8^x = 2$. (4 marks)

2016 1 (b) Solve the equation $2^{2x+1} + 5(2^x) - 3 = 0$.

(4 marks)

(c) (i) Given that $T = kp^{\left(\frac{h}{c}\right)}$, make c the subject of the formula.

(2 marks)

(ii) Solve the equation

$$\log(x+1) + \log(x-1) = 2\log(x+2).$$

(2 marks)

2017 1 (c) The values of the variables P and x in Table 1 obtained from an experiment are thought to obey a law of the form $P = Ax^{-k}$.

TABLE 1

x	1.58	2.51	3.98	6.30	10.0
P	121.5	110.6	106.2	99.1	93.8

(i) Use logarithms to reduce the equation to linear form.

(1 mark)

(ii) Using a suitable scale, plot the best fit line of the equation in (c) (i) on the graph paper provided on page 9. Use the space below to show your working.

(3 marks)

(iii) Hence, estimate the constants A and k.

(3 marks)

2018 1 (c) An equation relating V and t is given by $V = ka^t$ where k and a are constants.

(i) Use logarithms to derive an equation of the form $y = mx + c$ that can be used to find the values of k and a.

(2 marks)

(ii) If a graph of y versus x from the equation in Part (c) (i) is plotted, a straight line is obtained. State an expression for the gradient of the graph.

(1 mark)

CORDINATE (& CIRCLE) GEOMETRY

SPEC 3.(a)
'10

(i) Write the equation of the circle C, with centre $(-1, 2)$ and radius $\sqrt{13}$ units.

[1 mark]

(ii) Find the equation of the tangent to the circle C at the point $P(2,4)$.

[4 marks]

2012 3(a) The equation of a circle is given by $x^2 + y^2 - 4x + 6y = 87$.

(i) A line has equation $x + y + 1 = 0$. Show that this line passes through the centre of the circle. (3 marks)

(ii) Find the equation of the tangent to the circle at the point $A(-6, 3)$. (4 marks)

2013 3.(a)

(i) A circle, C, has centre with coordinates $A(2,1)$ and passes through the point $B(10,7)$.

Express the equation of the circle in the form $x^2 + y^2 + hx + gy + k = 0$, where h, g and k are integers to be determined. (3 marks)

(ii) The line l is a tangent to the circle C at the point B. Find an equation for l. (3 marks)

2014 3.(a)

(i) Determine the value of k such that the lines $x + 3y = 6$ and $kx + 2y = 12$ are perpendicular to each other. (3 marks)

(ii) A circle of radius 5 cm has as its centre the point of intersection of the two perpendicular lines in (i). Determine the equation for this circle. (3 marks)

2015 3.(a) The equation of a circle is given by $x^2 + y^2 - 12x - 22y + 152 = 0$.

(i) Determine the coordinates of the centre of the circle. (2 marks)

(ii) Find the length of the radius. (1 mark)

(iii) Determine the equation of the normal to the circle at the point $(4, 10)$. (3 marks)

2016 **3.(a)**   (i)   The points $M(3, 2)$ and $N(-1, 4)$ are the ends of a diameter of circle C. Determine the equation of circle C.

(5 marks)

(ii)   Find the equation of the tangent to the circle C at the point $P(-1, 6)$.

(3 marks)

2017 **3.(a)**   A circle C has an equation $x^2 + y^2 + 4x - 2y - 20 = 0$.

(i)   Express the equation in the form $(x + f)^2 + (y + g)^2 = r^2$.

(2 marks)

(ii)   State the coordinates of the centre and the value of the radius of circle C.

(2 marks)

(iii)   Determine the points of intersection of circle C and the equation $y = 4 - x$

(4 marks)

2018. **3. (a)**   Determine the equation of the circle that has centre $(5, -2)$, and passes through the origin.

(3 marks)

(b)   Determine whether the following pair of lines is parallel.

$x + y = 4$
$3x - 2y = -3$

(2 marks)

2019. **3. (a)**   A circle with centre $(1, -1)$ passes through the point $(4, 3)$.

(i)   Calculate the radius of the circle.

(2 marks)

(ii)   Write the equation of the circle in the form $x^2 + y^2 + 2fx + 2gy + c = 0$.

(2 marks)

(iii)   Determine the equation of the tangent to the circle at the point $(4, 3)$.
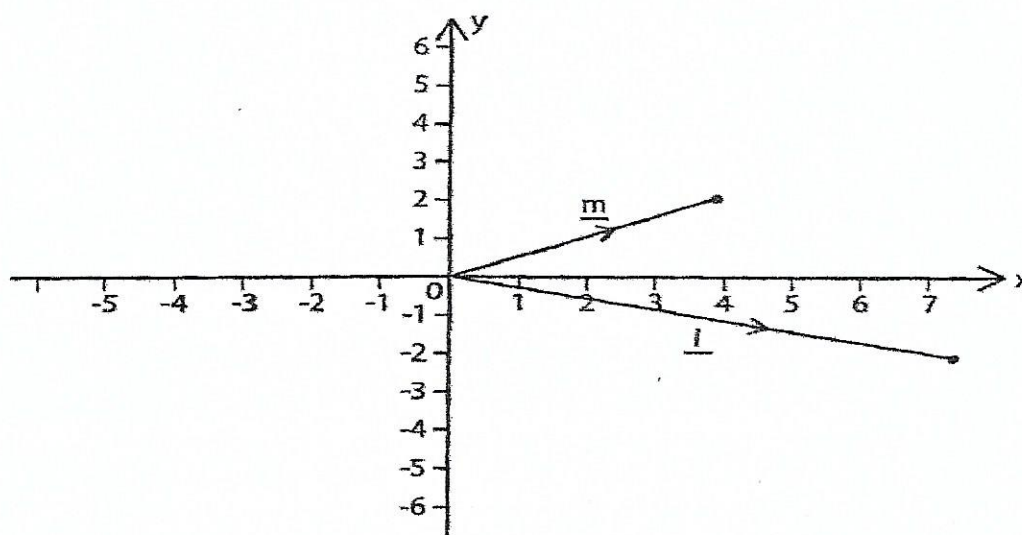
(3 marks)

VECTORS

SPEC 3.(b) The position vector of two points, A and B, relative to a fixed origin O, are
$3t\mathbf{i} + 2t\mathbf{j}$ and $4\mathbf{i} - 2t\mathbf{j}$ respectively, where $t > 0$.

Find the value of t such that $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are perpendicular.

[4 marks]

(c) The points L and M referred to a fixed origin O are represented by the vectors $\mathbf{l} = 7\mathbf{i} - 2\mathbf{j}$ and
$\mathbf{m} = 4t\mathbf{i} + 2\mathbf{j}$ respectively, as shown in the diagram below.

Find the unit vector in the direction of $\overrightarrow{LM}$.

[3 marks]

2012 3.(b) Given $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{AP} = \frac{1}{2}\overrightarrow{OA}$,

where $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

(i) Write $\overrightarrow{BP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (2 marks)

(ii) Find $|\overrightarrow{BP}|$. (3 marks)

2013 3.(b) The position vectors of two points, P and Q, relative to a fixed origin, O, are $10i - 8j$ and $\lambda i + 10j$ respectively, where λ is a constant.

Find the value of λ such that $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are perpendicular. **(3 marks)**

(c) The position vectors of A and B with respect to a fixed origin, O, are given by $\overrightarrow{OA} = -2i + 5j$ and $\overrightarrow{OB} = 3i - 7j$ respectively.

Find the unit vector in the direction of AB. **(3 marks)**

2014 3.(b) RST is a triangle in the coordinate plane. Position vectors R, S, and T relative to an origin, O, are $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ respectively.

    (i) Show that $T\hat{R}S = 90°$. **(4 marks)**

    (ii) Determine the length of the hypotenuse. **(2 marks)**

    [Hint: A rough drawing of RST might help].

2015 3.(b) The position vectors of two points, A and B, relative to an origin O, are such that

    $OA = 3i - 2j$ and $OB = 5i - 7j$. Determine

    (i) the unit vector AB **(3 marks)**

    (ii) the acute angle AOB, in degrees, to one decimal place. **(3 marks)**

2016 3.(b) The position vector of two points A and B, relative to a fixed origin, O, are $\bar{a}$ and $\bar{b}$

    respectively, where $\bar{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, and $\bar{b} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$. P lies on $\overrightarrow{AB}$ such that $\overrightarrow{PB} = \frac{1}{4}\overrightarrow{AB}$.

    Find the coordinates of $\overrightarrow{OP}$.

(4 marks)

2017 3.(b) Given that $p = 2i + 3j$ and $q = i + 5j$, determine

    (i) the product of the two vectors, p and q

(1 mark)

    (ii) the angle between the two vectors. **(3 marks)**

2018 3.(c) The position vectors of two points, A and B, relative to a fixed origin, O, are given by $\overrightarrow{OA} = 2\mathbf{i} + \mathbf{j}$ and $\overrightarrow{OB} = 3\mathbf{i} - 5\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ represent the unit vectors in the x and y directions respectively. Calculate

    (i)   the magnitude of $\overrightarrow{AB}$ **(4 marks)**

    (ii)   the angle $A\hat{O}B$, giving your answer to the nearest whole number.   **(3 marks)**

2019 3. (b) Two vectors p and q are such that $\mathbf{p} = 8\mathbf{i} + 2\mathbf{j}$ and $\mathbf{q} = \mathbf{i} - 4\mathbf{j}$.

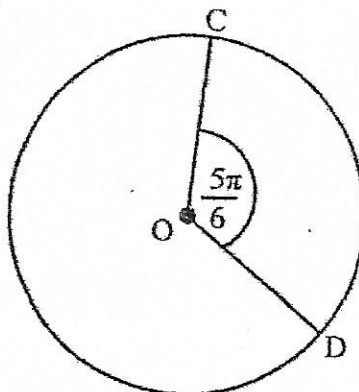   (i)   Calculate p.q.   **(2 marks)**

    (ii)   State the angle between the two vectors p and q.   **(1 mark)**

    (c)   The position vector $\mathbf{a} = 4\mathbf{i} - 7\mathbf{j}$. Find the unit vector in the direction of a.   **(2 marks)**

TRIGONOMETRY I

Circular Measure

SPEC 4(a) '10

The diagram below shows a circle of centre O and radius 6 cm. The sector COD subtends the angle $\frac{5\pi}{6}$ at the centre.

Working in **radians**, calculate, giving your answers in terms of π,

(i)   the length of the minor arc CD

[1 mark]

(ii)   the area of the minor sector OCD

[2 marks]

2012 4.(a)

The diagram shows a sector of a circle centre O with an adjoining square. The radius of the circle is 4 m.
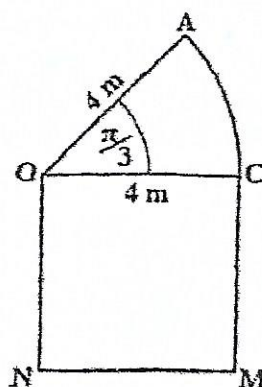
If the sector AOC subtends an angle $\frac{\pi}{3}$ at O, calculate, giving your answer in terms of π

(i)   the area of the shape $OACMN$

(ii)   the perimeter of the shape $OACMN$.

(5 marks)

2013   4.(a)
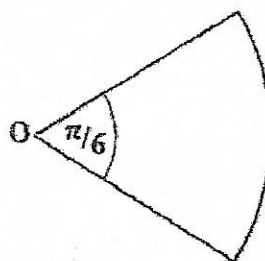
The diagram shows a sector cut from a circle of centre O. The angle at O is $\frac{\pi}{6}$. If the perimeter of the sector is $\frac{5}{6}(12 \div \pi)$ cm, what is its area?   **(4 marks)**

2014   4.(a)   Figure 1 shows the sector OAB of a circle with centre O, radius 9 cm and angle 0.7 radians.
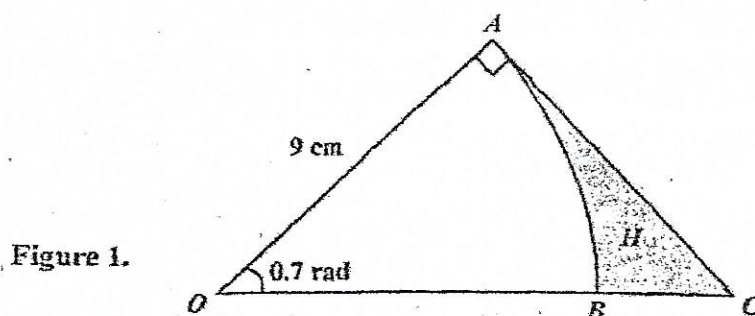
Figure 1.

(i)   Find the area of the sector OAB.   **(2 marks)**

(ii)   Hence, find the area of the shaded region, H.   **(4 marks)**

2015 4.(a)   The following diagram shows a circle of radius $r = 4$ cm, with centre O and sector AOB which subtends an angle, $\theta = \frac{\pi}{6}$ radians at the centre.
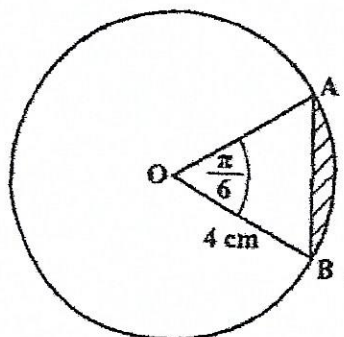
If the area of the triangle AOB $= \frac{1}{2} r^2 \sin \theta$, then calculate the area of the shaded region.
(4 marks)

2016 4. (a) The following diagram (not drawn to scale) shows two sectors, AOB and DOC. OB and OC are x cm and $(x + 2)$ cm respectively and angle $AOB = \theta$.
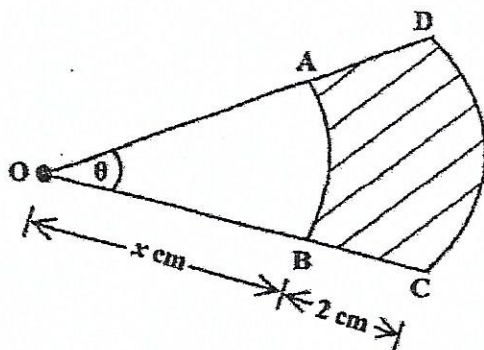
If $\theta = \dfrac{2\pi}{9}$ radians, calculate the area of the shaded region in terms of x.

(4 marks)

2017 4. (a) Figure 1 shows a plot of land, ABCD (**not drawn to scale**). Section ABC is used for building and the remainder for farming. The radius BC is 10 m and angle BCD is a right angle.
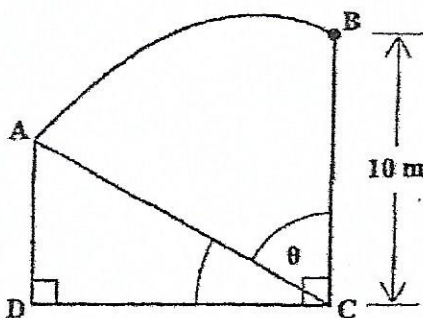
Figure 1

(i) If the building space is $\dfrac{50\pi}{3}$ m², calculate the angle ACB in radians. **(1 mark)**

(ii) Working in radians, calculate the area used for farming. **(4 marks)**

2018 4. (a) A wire in the form of a circle with radius 4 cm is reshaped in the form of a sector of a circle with radius 10 cm. Determine, in radians, the angle of the sector, giving your answer in terms of π.

(4 marks)

2019 4. (a) A compass is used to draw a sector of radius 6 cm and area 11.32 cm².

(i) Determine the angle of the sector in radians. **(3 marks)**

(ii) Calculate the perimeter of the sector. **(2 marks)**

TRIGONOMETRY II

Equations, Identities & Small Angles

SPEC 4 (b) '10

(i)   Given that $\sin \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$, where x is acute, show that

$$\sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\sin x - \cos x).$$

[2 marks]

(ii)   Using the fact that $\sin \dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$ and $\cos \dfrac{\pi}{3} = \dfrac{1}{2}$, find the exact value of

$\sin \dfrac{\pi}{12}$ showing <u>ALL</u> steps in your working.

[3 marks]

(c)   Prove the identity $\left(\tan \theta - \dfrac{1}{\cos \theta}\right)^2 \equiv -\dfrac{\sin \theta - 1}{\sin \theta + 1}$.

[4 marks]

2012 4.(b)   Given that $\sin \dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$, $\cos \dfrac{\pi}{3} = \dfrac{1}{2}$ and $\sin \dfrac{\pi}{4} = \cos \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$, evaluate without using

calculators, the exact value of $\cos \dfrac{7\pi}{12}$.   (3 marks)

(c)   Prove the identity

$$\frac{1}{\sec \theta + \tan \theta} \equiv \frac{1 - \sin \theta}{\cos \theta}.$$

(4 marks)

2013 4 (b)   Solve the equation $2 \cos^2\theta + 3 \sin \theta = 0$ for $0 \le \theta \le 360°$.   (5 marks)

(c)   Given that $\tan (\theta - \alpha) = \dfrac{1}{2}$ and that $\tan \theta = 3$, use the appropriate compound angle
formula to find the value of the acute angle α.   (3 marks)

2014 4.(b) Given that $\sin \dfrac{\pi}{6} = \dfrac{1}{2}$ and $\cos \dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$, show that

$$\cos\left(x + \dfrac{\pi}{6}\right) = \dfrac{1}{2}\,(\sqrt{3}\,\cos x - \sin x), \text{ where } x \text{ is acute.} \qquad \textbf{(2 marks)}$$

(c) Prove the identity $\left(\dfrac{\tan \theta\ \sin \theta}{1 - \cos \theta}\right) \equiv 1 + \dfrac{1}{\cos \theta}$. **(4 marks)**

2015 4.(b) Solve the following equation, giving your answer correct to one decimal place.

$$8 \sin^2 \theta = 5 - 10 \cos \theta, \text{ where } 0^\circ \leq \theta \leq 360^\circ \qquad \textbf{(4 marks)}$$

(c) Prove the identity

$$\dfrac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta} \equiv \tan \theta. \qquad \textbf{(4 marks)}$$

2016 4.(b) Given that $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$ and $\sin 45^\circ = \dfrac{\sqrt{2}}{2}$, without the use of a calculator, evaluate

$\cos 105^\circ$, in surd form, giving your answer in the simplest terms.

(5 marks)

(c) Prove that the identity $\dfrac{\sin(\theta + \alpha)}{\cos \theta \cos \alpha} \equiv \tan \theta + \tan \alpha$.

(3 marks)

2017 4.(b) Given that

$$\sin \dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}, \qquad \cos \dfrac{\pi}{3} = \dfrac{1}{2} \text{ and } \qquad \sin \dfrac{\pi}{4} = \cos \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2},$$

show without using a **calculator** that

$$\dfrac{\cos\left(\dfrac{\pi}{4} - \dfrac{\pi}{3}\right)}{\sin \dfrac{2\pi}{3}} = \dfrac{\sqrt{2} + \sqrt{6}}{2\sqrt{3}}.$$

(4 marks)

(c) Prove the identity

$$1 - \dfrac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta.$$

(3 marks)

2018 4.(b) Solve the equation $\sin^2 \theta + 3 \cos 2\theta = 2$ for $0 \le \theta \le \pi$. Give your answer(s) to 1 decimal place.

(4 marks)

(c) Prove the identity $\dfrac{1}{1 - \sin x} - \dfrac{1}{1 + \sin x} \equiv \dfrac{2 \tan x}{\cos x}$.

(4 marks)

2019 4.(b) A right-angled triangle XYZ has an angle, θ, where $\sin \theta = \dfrac{\sqrt{5}}{5}$.

Without evaluating θ, calculate the exact value (in surd form if applicable) of

(i) $\cos \theta$

(2 marks)

(ii) $\sin 2\theta$.

(2 marks)

(c) Show that $\tan^2 \theta + 1 = \dfrac{1}{\cos^2 \theta}$.

(3 marks)

Introductory Calculus

DIFFERENTIATION I

SPEC '10 5.(a)   Differentiate the following with respect to x, simplifying your result as far as possible

$$(5-2x)(1+x)^4$$

[4 marks]

2012 5.(a)   Differentiate the following expression with respect to x, simplifying your answer.

$$\frac{3x+4}{x-2}$$

(4 marks)

2013 5.(b)   Differentiate $y = (5x+3)^3 \sin x$ with respect to x, simplifying your result as far as possible.

(4 marks)

2015 5.(a)   Differentiate the following expression with respect to x, simplifying your answer.

$$(2x^2+3)\sin 5x$$

(4 marks)

2016 5.(a)   Find $\frac{dy}{dx}$ given that $y = \sqrt{5x^2-4}$, simplifying your answer.

(4 marks)

2017 5.(a)   Differentiate the expression $(1+2x)^3(x+3)$ with respect to x, simplifying your answer.

(4 marks)

2018 5.(b)   Differentiate $y = 2x\sqrt{(4-8x)}$ with respect to x, simplifying your answer.

(4 marks)

2019 5.(b)   Differentiate $\sqrt[3]{(2x+3)^7}$ with respect to x, giving your answer in its simplest form.

(3 marks)

DIFFERENTIATION II

Gradients, Tangents, Normals, Stationary Points

SPEC 5.(b) '10 The point P lies on the curve $y = x^2$. The value of x at P is -2.

Find the equation of the tangent to the curve at P.

[4 marks]

SPEC 5.(c) '10 Find the stationary points on the curve $f(x) = 2x^3 - 9x^2 + 12x$ and distinguish their nature.

[6 marks]

2012 5.(b) The point $P(2, 10)$ lies on the curve $y = 3x^2 + 5x - 12$. Find equations for

(i) the tangent to the curve at P

(ii) the normal to the curve at P.

(5 marks)

2013 5. (a) Given that $y = x^3 - 3x^2 + 2$. Find

(i) the coordinates of the stationary points of y

(5 marks)

(ii) the second derivative of y and hence determine the nature of EACH of the stationary points.

(5 marks)

2014 5.(a) The equation of a curve is $y = 3 + 4x - x^2$. The point $P(3, 6)$ lies on the curve.

Find the equation of the tangent to the curve at P, giving your answer in the form

$ax + by + c = 0$, where $a, b, c, \in \mathbb{Z}$.

(4 marks)

2014. 5.(b) Given that $f(x) = 2x^3 - 9x^2 - 24x + 7$.

(i) Find ALL the stationary points of $f(x)$.

(5 marks)

(ii) Determine the nature of EACH of the stationary points of $f(x)$.

(5 marks)

2015 5.(b) (i) Find the coordinates of all the stationary points of the curve $y = x^3 - 5x^2 + 3x + 1$.

(3 marks)

(ii) Determine the nature of EACH point in (i) above.

(2 marks)

2016 5.(b) The point P (1, 8) lies on the curve with equation $y = 2x (x + 1)^2$. Determine the equation of the normal to the curve at the point P.

(5 marks)

(c) Obtain the equation for EACH of the two tangents drawn to the curve $y = x^2$ at the points where $y = 16$.

(5 marks)

2017 5.(b) The point P (–2, 0) lies on the curve $y = 3x^3 + 2x^2 - 24x$. Determine the equation of the normal to the curve at point P.

(5 marks)

2018 5 (a) Given that $y = x^3 + 2x^2 - 1$, determine

(i) the coordinates of the stationary points

(5 marks)

(ii) the nature of EACH stationary point.

(5 marks)

2019 5. (a) The stationary points of a curve are given by (5, $11\frac{2}{3}$) and (3, 15).

(i) Derive an expression for $\frac{dy}{dx}$.

(2 marks)

(ii) Determine the nature of the stationary points.

(5 marks)

(iii) Determine the equation of the curve.

(4 marks)

DIFFERENTIATION III

Rates of Change

2012 5.(c) The length of the side of a square is increasing at a rate of 4 cms^{-1}. Find the rate of increase of the area when the length of the side is 5 cm. (5 marks)

2015 5.(c) A spherical balloon of volume $V = \frac{4}{3} \pi r^3$ is being filled with air at the rate of 200 cm^3 s^{-1}.

Calculate, in terms of π, the rate at which the radius is increasing when the radius of the balloon is 10 cm. (5 marks)

2017 5.(c) Water is poured into a cylindrical container of radius 15 cm. The height of the water increases at a rate of 2 cms^{-1}. Given that the formula for the volume of a cylinder is $\pi r^2 h$, determine the rate of increase of the volume of water in the container in terms of π.

(5 marks)

INTEGRATION I

SPEC '10 6.(a)   Evaluate $\int_{1}^{2}(3x-1)^2\,dx$.   [4 marks]

(b)   Evaluate $\int_{0}^{\frac{\pi}{2}}(5\sin x - 3\cos x)\,dx$.   [4 marks]

2012 6.(a)   Evaluate $\int_{1}^{2}(16-7x)^3\,dx$.   (4 marks)

2013 6.(a)   Find $\int(5x^2+4)\,dx$.   (2 marks)

(b)   Evaluate $\int_{0}^{\frac{\pi}{2}}(3\sin x - 5\cos x)\,dx$.   (4 marks)

2014 6.(a)   Evaluate $\int_{2}^{4}x(x^2-2)\,dx$.   (4 marks)

(b)   Evaluate $\int_{0}^{\frac{\pi}{3}}(4\cos x + 2\sin x)\,dx$, leaving your answer in surd form.   (4 marks)

2015 6.(a)   Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\cos 3\theta\,d\theta$.   (4 marks)

2016 6.(a)   (i)   Find $\int(3\cos\theta - 5\sin\theta)\,d\theta$.   (3 marks)

(ii)   Evaluate $\int_{1}^{3}\left[\frac{2}{x^2}-3+2x^3\right]dx$.   (4 marks)

2017 6.(a)   Show that $\int_0^{\frac{\pi}{4}} (\sin x + 4 \cos x)\, dx = \dfrac{3\sqrt{2}+2}{2}$.

(4 marks)

2017 6. (c)   Evaluate $\int_1^2 (4-x)^2\, dx$.

(3 marks)

2019 6. (a)   Integrate $3 \cos x + 2 \sin x$.   (2 marks)

(b)   Evaluate $\int_1^4 \dfrac{2\sqrt{x}}{x}\, dx$ ·

(4 marks)

INTEGRATION II

Plane Areas, Volume of Solid of Revolution

SPEC 6.(c)
'10

A curve passes through the point P $\left(0, \dfrac{7}{2}\right)$ and is such that $\dfrac{dy}{dx} = 2 - x$.

(i) Find the equation of the curve. [3 marks]

(ii) Find the area of the finite region bounded by the curve, the x-axis,

the y-axis and the line x=5.

[3 marks]

2012 6.(b)
The point Q (4, 8) lies on a curve for which $\dfrac{dy}{dx} = 3x - 5$. Determine the equation of the curve. (3 marks)

(c) Calculate the area between the curve $y = 2\cos x + 3\sin x$ and the x-axis from $x = 0$ to $\dfrac{\pi}{3}$.

(3 marks)

(d) Calculate the volume of the solid formed when the area enclosed by the curve $y = x^2 + 2$ and the x-axis, from $x = 0$ to $x = 3$, is rotated through 360° about the x-axis.

[Leave your solution in terms of π]. (4 marks)

2013 6.(c)
A curve passes through the points P(0, 8) and Q(4, 0) and is such that $\dfrac{dy}{dx} = 2 - 2x$.

Find the area of the finite region bounded by the curve in the first quadrant.

(8 marks)

2014 6.(c)
A curve passes through the point P (2, −5) and is such that $\dfrac{dy}{dx} = 6x^2 - 1$.

(i) Determine the equation of the curve. (3 marks)

(ii) Find the area of the finite region bounded by the curve, the x-axis, the line $x = 3$ and the line $x = 4$. (3 marks)

2015 6.(b) A curve has an equation which satisfies $\frac{dy}{dx} = kx(x-1)$ where k is a constant.

Given that the value of the gradient of the curve at the point $(2, 3)$ is 14, determine

    (i)   the value of k **(2 marks)**

    (ii)   the equation of the curve.   **(4 marks)**

(c) Calculate, in terms of π, the volume of the solid formed when the area enclosed by the curve $y = x^2 + 1$ and the x-axis, from $x = 0$ to $x = 1$, is rotated through $360°$ about the x-axis.

                                                                         (4 marks)

2016 6.(b) The following figure shows the finite region R bounded by the lines $x = 1$, $x = 2$ and the arc of the curve $y = (x-2)^2 + 5$.
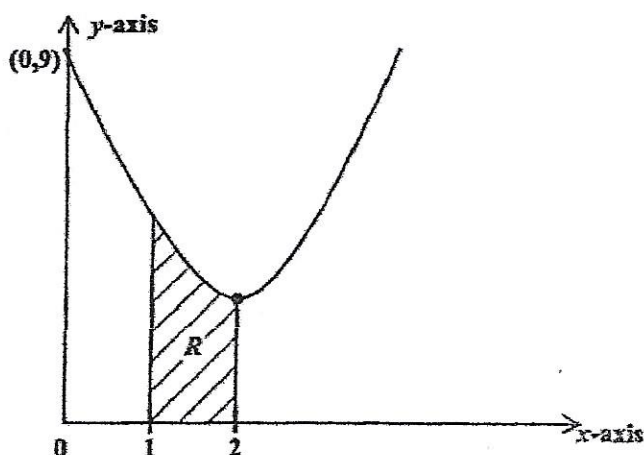
Calculate the area of the region R.

                                                                         (4 marks)

(c) The point $P(1, 2)$ lies on the curve which has a gradient function given by $\frac{dy}{dx} = 3x^2 - 6x$. Find the equation of the curve.

                                                                         (3 marks)

2017 6.(b) Determine the equation of a curve whose gradient function $\frac{dy}{dx} = x + 2$, and which passes through the point $P(2, 3)$.

                                                                         (3 marks)

2017 6.(d) Calculate the volume of the solid formed when the area enclosed by the straight line

$y = \frac{x}{2}$ and the x-axis for $x = 0$ to $x = 6$ is rotated through 2π about the x-axis.

                                                                         (4 marks)

2018 6 (a) Show, using integration, that the finite area of the curve $y = \sin x$ in the first quadrant bounded by the line $x = \dfrac{4\pi}{9}$ is smaller than the finite region of $y = \cos x$ in the same quadrant and bounded by the same line.

(6 marks)

(b) The finite region in the first quadrant bounded by the curve $y = x^2 + x + 3$, the x-axis and the line $x = 4$ is rotated completely about the x-axis. Determine the volume of the solid of revolution formed.

(4 marks)

(c) A curve which has a gradient of $\dfrac{dy}{dx} = 3x - 1$ passes through the point $A(4, 1)$. Find the equation of the curve.

(4 marks)

2019 6(c) The point $(2, 4)$ lies on the curve whose gradient is given by $\dfrac{dy}{dx} = -2x + 1$.

Determine

(i) the equation of the curve

(4 marks)

(ii) the area under the curve in the finite region in the first quadrant between 0 and 3 on the x-axis.

(4 marks)

Probability and Statistics

STATISTICS I

Probability Theory

SPEC '10 7.(a) In a Lower Sixth Form there are 43 students who are studying either Statistics or Physics or both Statistics and Physics. 28 students study Statistics and 19 students study Physics. If a student is selected at random, what is the probability that he/she is studying

(i)   both Statistics and Physics,   [3 marks]

(ii)   Physics only.   [2 marks]

(b)   A tetrahedral die has four faces. Two fair tetrahedral dice, one red and one blue, have faces numbered 0, 1, 2, and 3 respectively. The dice are rolled and the numbers face down on the two dice are recorded. R is the score on the red die and B is the score on the blue die.

(i)   Find $P(R = 3 \text{ and } B = 0)$.   [2 marks]

The random variable T is R multiplied by B.

(ii)   Complete the diagram below to represent the sample space that shows all the possible values of T

B\R	0	1	2	3
3				
2		2		
1	0			
0				

Sample space diagram of T

[3 marks]

The table below gives the probability of each possible value of t.

t	0	1	2	3	4	6	9
$P(T = t)$	a	$\dfrac{1}{16}$	$\dfrac{1}{8}$	b	c		

(iii) Find the values of a, b and c.

[3 marks]

2012 7.(a) A survey carried out in a town revealed that 25% of the households surveyed owned a laptop computer and 70% owned a desktop computer. In addition, it was found that 12% owned both a laptop and a desktop computer.

If a sample of households from the town is selected at random, determine the proportion that own NEITHER a laptop NOR a desktop computer.

(4 marks)

(b) A bag contains 4 red marbles, 3 black marbles and 3 blue marbles. Three marbles are drawn at random without replacement from the bag.

Find the probability that the marbles

(i) drawn are ALL of the SAME colour (3 marks)

(ii) contain EXACTLY 1 red marble. (3 marks)

(c) The probability of hiring a taxi from garage A, B or C is 0.3, 0.5 and 0.2 respectively. The probability that the taxi ordered will be late from A is 0.07, from B is 0.1 and from C is 0.2.

(i) Illustrate this information on a tree diagram showing the probability on all branches. (3 marks)

(ii) A garage is chosen at random, determine the probability that

a) the taxi will arrive late (3 marks)

b) the taxi will come from garage C given that it is late. (4 marks)

2013 7.(a) Of the persons buying petrol at a service station, 40 per cent are females. Of the females, 30 per cent pay for their petrol with cash, and of the males, 65 per cent pay for their petrol with cash.

(i) Copy and complete the following tree diagram, by putting in all the missing probabilities, to show this information. (2 marks)
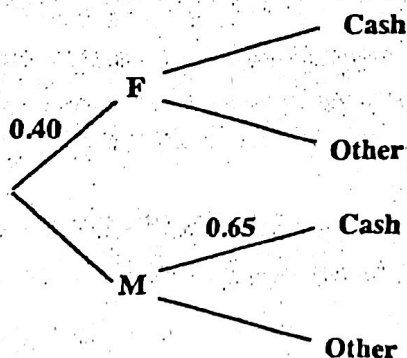
(ii) What is the probability that a customer pays for petrol with cash? (3 marks)

(iii) Determine which is the more likely event:

Event T: Customer is female, GIVEN that the petrol is paid WITH cash.
Event V: A male customer does NOT pay for petrol with cash. (4 marks)

7.(a) There are 60 students in the sixth form of a certain school. Mathematics is studied by 27 of them, Biology by 20 of them and 22 students study neither Mathematics nor Biology. If a student is selected at random, what is the probability that the student is studying

     (i)   both Mathematics and Biology?   **(3 marks)**

     (ii)   Biology only?   **(2 marks)**

(b) Two ordinary six-sided dice are thrown together. The random variable S represents the sum of the scores of their faces landing uppermost.

     (i)   Copy and complete the sample space diagram below.

6			9			
5		7				
4						10
3					8	
2				6		
1	2					
	1	2	3	4	5	6

Sample space diagram of S

                                                      (1 mark)

     (ii)   Find

         a) $P(S > 9)$ **(2 marks)**

         b) $P(S \leq 4)$.   **(1 mark)**

     (iii)   Let D be the difference between the scores of the faces landing uppermost. The table below gives the probability of each possible value of d.

d	0	1	2	3	4	5
$P(D=d)$	$\frac{1}{6}$	a	$\frac{2}{9}$	b	$\frac{1}{9}$	c

Find the values of a, b and c.   **(3 marks)**

2015 7.(a) There are three traffic lights that a motorist must pass on the way to work. The probability that the motorist has to stop at the first traffic light is 0.2, and that for the second and third traffic lights are 0.5 and 0.8 respectively. Find the probability that the motorist has to stop at

    (i)   ONLY ONE one traffic light   **(4 marks)**

    (ii)   AT LEAST TWO traffic lights.   **(4 marks)**

2015 7(c) Research in a town shows that if it rains on any one day then the probability that it will rain the following day is 25%. If it does not rain one day then the probability that it will rain the following day is 12%. Starting on a Monday and given that it rains on that Monday:

    (i)   Draw a probability tree diagram to illustrate the information, and show the probability on ALL of the branches.   **(4 marks)**

    (ii)   Determine the probability that it will rain on the Wednesday of that week.   **(4 marks)**

2017 7(a) The probability of a final-year college student receiving a reply for an internship programme from three accounting firms, Q, R and S, is 0.55, 0.25 and 0.20 respectively. The probability that the student receives a reply from firm Q and is accepted is 0.95. The probability that a student receives a reply from firms R and S and is accepted is 0.30 for each of them.

    (i)   Draw a tree diagram to illustrate the information above.

        (4 marks)

    (ii)   Determine the probability that the student will be accepted for an internship programme.

        (4 marks)

2017 7(c) A school cafeteria sells 20 chicken patties, 10 lentil patties and 25 saltfish patties daily. On a particular day, the first student ordered 2 patties but did not specify the type. The vendor randomly selects 2 patties.

    (i)   Calculate the probability that the first patty selected was saltfish.

        (4 marks)

    (ii)   Given that the first patty was saltfish, calculate the probability that the second patty was NOT saltfish.

        (2 marks)

2016 **7. (b)** Two events, A and B, are such that $P(A) = 0.5$, $P(B) = 0.8$ and $P(A \cup B) = 0.9$.

    (i)   Determine $P(A \cap B)$.

                        (2 marks)

    (ii)   Determine $P(A \mid B)$.

                        (2 marks)

    (iii)   State, giving a reason, whether or not A and B are independent events.

                        (2 marks)

    (c)   A bag contains 3 red balls, 4 black balls and 3 yellow balls. Three balls are drawn at random with replacement from the bag. Find the probability that the balls drawn are all of the same colour.

                        (4 marks)

2018. **8. (b)** Insecticides A, B or C are applied on lots Q, R and S. The same crop is planted on each lot and the lots are of the same size. The probability that a group of farmers will select A, B or C is 40%, 25% and 35% respectively. The probability that insecticide A is successful is 0.8, that B is successful is 0.65, and that C is successful is 0.95.

    (i)   Illustrate this information on a tree diagram showing ALL the probabilities on ALL branches.

                        (3 marks)

    (ii)   An insecticide is selected at random, determine the probability that it is unsuccessful.

                        (3 marks)

    (c)   A regular six-sided die is tossed 2 times.

    (i)   Calculate the probability of obtaining a 5 on the 2nd toss, given that a 5 was obtained on the 1st toss.

                        (2 marks)

    (ii)   Determine the probability that a 5 is obtained on both tosses.

                        (2 marks)

    (iii)   Explain why the answers in (c) (i) and (c) (ii) are different.

                        (1 mark)

2019 7.(b) A vendor has 15 apples on a tray: 5 red, 6 green and 4 yellow. A customer requests 3 apples but does NOT specify a colour.

Determine the probability that the apples chosen

(i)   contain one of EACH colour

(4 marks)

(ii)   are ALL of the same colour.

(4 marks)

STATISTICS II

Data Representation and Analysis

SPEC '10 7.(c) The number of cars parked on a local beachfront on each night in August last year is summarized in the following stem and leaf diagram.

```
1     0 5
2     1 2 4 8
3     0 3 3 3 4 7 8 8      Key:  1|0 means 10
4     1 1 3 5 8 8 8 9 9
5     2 3 6 6 7
6     2 3 4
```

(i)   Find the median and quartiles for these data.

[3 marks]

(ii)   Construct a box-and-whisker plot to illustrate these data and comment on the shape of the distribution.

[4 marks]

2013 7.(b) The marks obtained by 30 students on an English exam are given as

58	92	41	89	72	66	51	63	80	40
69	45	83	76	53	56	75	50	99	50
85	63	58	75	66	56	81	74	51	94

(i)   State ONE advantage of using a stem and leaf diagram versus a box and whiskers plot to display the data.   **(1 mark)**

(ii)   Construct a stem and leaf diagram to show the data.   **(3 marks)**

(iii)   Determine the median mark.   **(2 marks)**

(iv)   Calculate the semi inter-quartile range of the marks.   **(3 marks)**

(v)   Two students are chosen at random from the class.

Determine the probability that both scored less than 50 on the exam.

(2 marks)

2014 7.(c) The aptitude scores obtained by 51 applicants for a supervisory job are summarized in the following stem and leaf diagram.

5|1 means 51

```
3 | 1  5  9
4 | 2  4  6  8  9
5 | 1  3  3  5  6  7  9
6 | 0  1  3  3  3  5  6  8  8  9
7 | 1  2  2  2  4  5  5  5  6  8  8  8  9  9
8 | 0  1  2  3  5  8  8  9
9 | 0  1  2  6
```

(i) Find the median and quartiles for the data given. (4 marks)

(ii) Construct a box-and-whisker plot to illustrate the data given and comment on the distribution of the data. (4 marks)

2017 7.(b) Table 2 shows the length, in cm, of 20 spindles prepared by a carpenter to build a railing for an existing staircase.

TABLE 2

1.5	3.2	6.1	9.4	11.0	12.6	17.0	18.5	20.2	24.4
25.2	25.2	28.3	28.8	29.1	30.4	32.5	34.6	38.3	38.4

Determine

(i) the mean length

(2 marks)

(ii) the modal length

(1 mark)

(iii) the median length

(1 mark)

(iv) the interquartile range for the data.

(2 marks)

2015 7.(b) Use the data in the following table to estimate the mean of x.

x	5–9	10–14	15–19	20–24
f	8	4	10	3

(4 marks)

2016 7.(a) Use the data set provided below to answer the questions which follow.

15	16	18	18	20	21	22	22
22	25	28	30	30	32	35	40
41	52	54	59	60	65	68	75

(i) Construct a stem-and-leaf diagram to represent the given data.

(3 marks)

(ii) State an advantage of using the stem-and-leaf diagram to represent the given data.

(1 mark)

(iii) Determine the mode.

(1 mark)

(iv) Determine the median.

(2 marks)

(v) Determine the interquartile range.

(3 marks)

2018 7.(a) The number of runs scored by a cricketer for 18 consecutive innings is illustrated in the following stem-and-leaf diagram.

0	2	3	6	7	
1	0	3	5	8	9
2	4	4	6	8	
3	1	4	5		
4	5	7			

Key 0|6 means 6

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← 2018 7a. (i)   Determine the median score.   (2 marks)

(ii)   Calculate the interquartile range of the scores.

(iii)   In the space below, construct a box-and-whisker plot to illustrate the data and comment on the shape of the distribution.
                                                                                                            (4 marks)

2019 7. (a)   The weights, in kg, of students in a Grade 5 class are displayed in the following stem and leaf diagram.

                   Boys   Girls

                     9 8 | 2 | 8 8 9   Key:
                 9 9 9 7 3 | 3 | 2 2 3 5 8 8 8
             5 1 1 1 1 0 0 | 4 | 0 1 1 2   Boys 8|2 means 28 kg
                         1 | 5 | Girls 2|8 means 28 kg

(i)   State the number of students in the class.

....
                                                                                                            (1 mark)

(ii)   Construct ONE box-and-whisker plot for the entire Grade 5 class (boys and girls combined).
                                                                                                            (4 marks)

(iii)   The standard deviation of the weights of the boys is 5.53 kg.

Determine the standard deviation of the weights of the girls. Provide an interpretation of your answer for the girls compared to that given for the boys.
                                                                                                            (5 marks)

(iv)   Determine the number of students above the 20th percentile for this class.
                                                                                                            (2 marks)