# MATHEMATICS CSEC SUMMARY 2022

## Section 1 – Number Theory and Computation

**Sets of numbers**:

Natural numbers, N = {1, 2, 3,….}
Whole numbers, W = {0, 1, 2, 3, …..}
Integers, Z = { …, -2, -1, 0, 1, 2, …}

Rational numbers, Q = { $\frac{p}{q}$ , p and q are integers, q ≠ 0}

Irrational numbers, $\bar{Q}$ = { , $\sqrt{2}$ , $\pi$, ….}

Real numbers, R = Q ∪ $\bar{Q}$

**Significant figures rules**:

1. All non-zero numbers ARE significant
2. Zeros between two non-zero digits ARE significant
3. Leading zeros are NOT significant. .e.g. 0.0045 has 2 sig. fig.
4. Trailing zeros to the right of the decimal ARE significant. e.g. 45.00 has 4 sig. fig.
5. Trailing zeros in a whole number with decimal shown ARE significant.
6. Trailing zeros in a whole number with no decimal shown are NOT significant.

**Properties of numbers:**

a) Closure: If a,b ∈ $R$ then a*b ∈ $R$.
($b$) Associative: (x + y) + z = x + (y + z)
c) Commutative: x + y = y + x and x . y = y . x.
(d) Distributive: x . (y + z) = x . y. + x . z
e) Additive Identity: x + 0 = 0 + x = x.
(f) Multiplicative Identity: x . 1 = 1 . x = x
g) Additive Inverse: x + ( -x ) = ( - x ) + x = 0
(h) Multiplicative Inverse: x.$\left(\frac{1}{x}\right)$=$\left(\frac{1}{x}\right)$. x =1

**Ratios:**

A ratio of $\boldsymbol{a}$ **: b : c** implies that the fractions being shared are $\frac{\boldsymbol{a}}{\boldsymbol{a+b+c}}$ **:** $\frac{\boldsymbol{b}}{\boldsymbol{a+b+c}}$ **:** $\frac{\boldsymbol{c}}{\boldsymbol{a+b+c}}$

# Section 2 – Consumer Arithmetic

Discount = Selling Price x discount %

Sales Tax = $\frac{total\ sale\ tax}{Selling\ price} \times 100\%$

Profit = Selling Price – Cost price

Loss = Cost price – Selling price

Profit % = $\frac{SP-CP}{CP}$ x 100 %

Simple Interest = $\frac{Principal \times Rate \times time}{100}$

Compound Interest Formula, $A = P(1+i)^n$
A – final amount including principal
P – principal amount
i – interest rate per year
n – number of years

The Compound Interest formula can also be used to determine appreciation and depreciation.

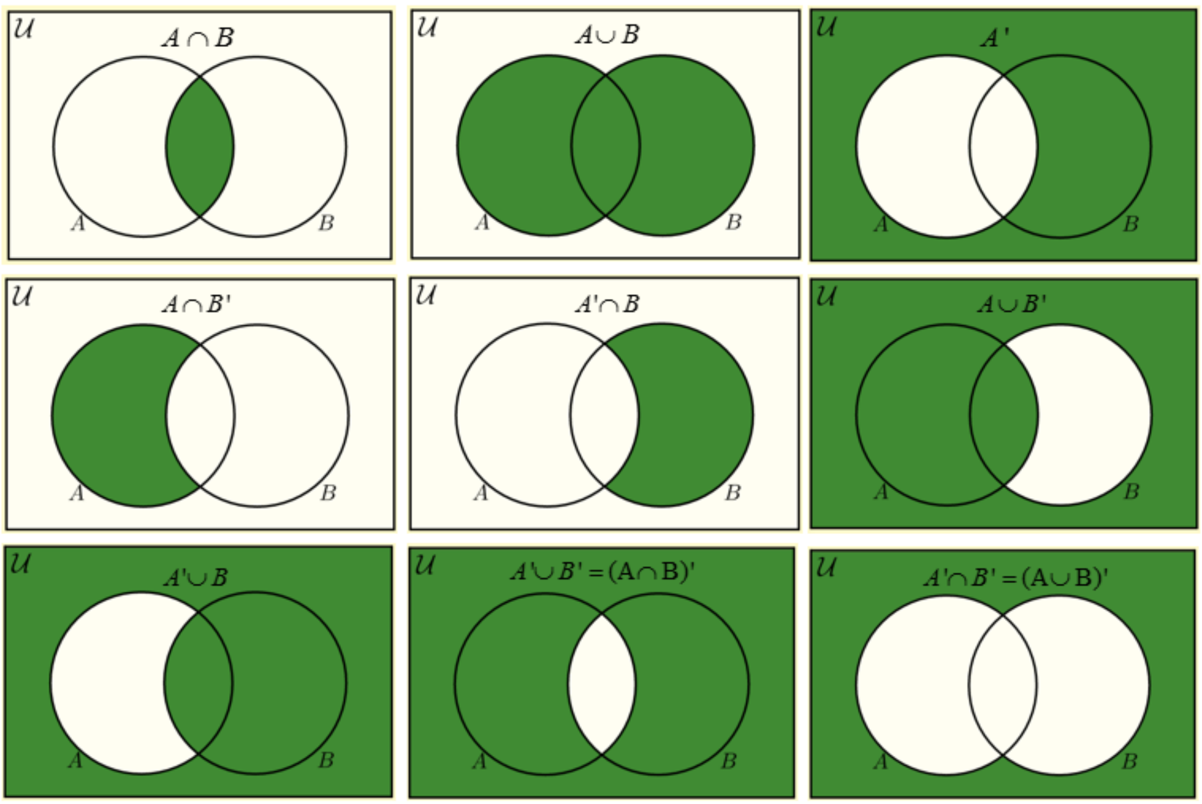
# Section 3 – Sets

## Set Theory Symbols

| Symbol | Name | Example | Explanation |
|---|---|---|---|
| { } | Set | $A=\{1,3\}$<br>$B=\{2,3,9\}$<br>$\mathrm{C}=\{3,9\}$ | Collection of objects |
| $\cap$ | Intersect | $A\cap B=\{3\}$ | Belong to both set A and set B |
| $\cup$ | Union | $A\cup B=\{1,2,3,9\}$ | Belong to set A or set B |
| $\subset$ | Proper Subset | $\{1\}\subset A$<br>$C\subset B$ | A set that is contained in another set |
| $\subseteq$ | Subset | $\{1\}\subseteq A$<br>$\{1,3\}\subseteq A$ | A set that is contained in or equal to another set |
| $\not\subset$ | Not a Proper Subset | $\{1.3\}\not\subset A$ | A set that is not contained in another set |
| $\supset$ | Superset | $B\supset C$ | Set B includes set C |
| $\in$ | Is a member | $3\in A$ | 3 is an element in set A |
| $\notin$ | Is not a member | $4\notin A$ | 4 is not an element in set A |

* $\mathrm{n}(\mathrm{A}\cup B)=n(A)+n(B)-n(A\cap B)$

<u>Venn diagrams</u>

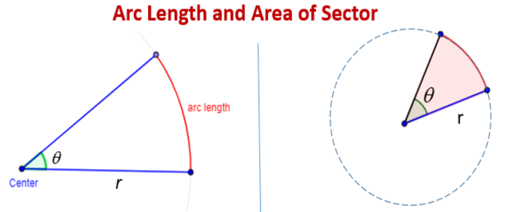


## Section 4 – Measurements

| Length | Mass |
|---|---|
| 10 mm = 1 cm | 1g = 1000mg |
| 100 cm = 1 m | 1kg = 1000g |
| 1000 mm = 1 m | 1kg = 2.2lbs |
| 1000 m = 1 km | 1lb = 16 ounces |

| Shape | Volume Formula | Surface Area Formula |
|---|---|---|
| Cube | $s^3$ where $s$ = length of the side | $6s^2$ where $s$ = length of the side |
| Cuboid | $lwh$ where $l$ = length, $w$ = width, $h$ = height | $2(lw + lh + wh)$ where $l$ = length, $w$ = width, $h$ = height |
| Prism | area of base × height | 2 × area of base + perimeter of base × height |
| Cylinder | $\pi r^2 h$ where $r$ = radius, $h$ = height | $2\pi r\,(r + h)$ where $r$ = radius, $h$ = height |
| Hollow Cylinder | $\pi h(R^2 - r^2)$ where $R$ = radius of the outer surface, $r$ = radius of the inner surface | $2\pi rh + 2\pi Rh + 2(\pi R^2 - \pi r^2)$ where $R$ = radius of the outer surface, $r$ = radius of the inner surface |
| Cone | $\frac{1}{3}\pi r^2 h$ where $r$ = radius, $h$ = height | $\pi r\,(r + s)$ where $r$ = radius, $s$ = slant height |
| Pyramid | $\frac{1}{3}Bh$ where B = area of base, h = height | Any pyramid = area of base + area of each of the lateral faces. Regular pyramid = area of base + $\frac{1}{2}ps$ where $p$ = perimeter of the base, $s$ = slant height. Square pyramid = $b^2 + 2bs$ where $b$ = length of the base, $s$ = slant height |
| Sphere | $\frac{4}{3}\pi r^3$ where $r$ = radius | $4\pi r^2$ where $r$ = radius |
| Hemisphere | $\frac{2}{3}\pi r^3$ where $r$ = radius | $3\pi r^2$ where $r$ = radius |

**Arc Length and Area of Sector**




If $\theta$ is measured in degrees then arc length $= \frac{\theta}{360^\circ} \times 2\pi r$

If $\theta$ is measured in degrees then area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

Speed $= \frac{distnace}{time}$

Units: $ms^{-1}$ or $kmh^{-1}$

Distance = speed x time

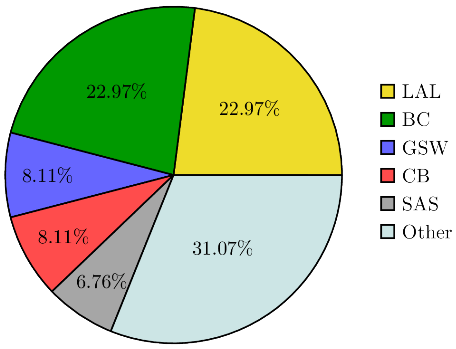
Time $= \frac{distance}{speed}$

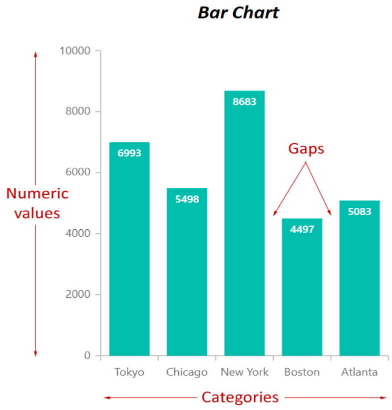
# Section 5 – Statistics

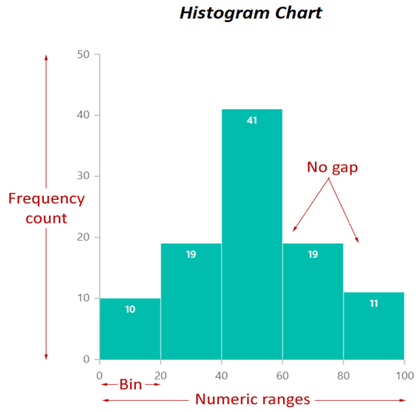
## Basic definitions

- Population: The entire group being investigated
- Sample: A subset of the population
- Discrete data: Specific values only
- Continuous data: Range of values
- Raw data: Unordered info
- Mean: Average value $= \frac{\sum x}{n}$ *(ungrouped)* or $\frac{\sum fx}{\sum f}$ *(grouped data)*
- Median: Middle value from a set of ordered values
- Mode: Most frequent value
- Probability: Chances of an event occurring
- Standard deviation: Gives a spread of the data. (*how far away from mean)*

Types of statistical charts:




To plot histograms, we need the class boundaries as shown below:

| Class Limits | Class boundaries | Tally | Frequency |
|---|---|---|---|
| 318 - 335 | 317.5 - 335.5 | \|\|\|\| | 4 |
| 336 - 353 | 335.5 - 353.5 | 𝍸 | 5 |
| 354 - 371 | 353.5 - 371.5 | \|\| | 2 |
| 372 - 389 | 371.5 - 389.5 | \|\|\| | 3 |
| 390 - 407 | 389.5 - 407.5 | \|\|\| | 3 |
| 408 - 425 | 407.5 - 425.5 | \|\| | 2 |
| 426 - 443 | 425.5 - 443.5 | \| | 1 |

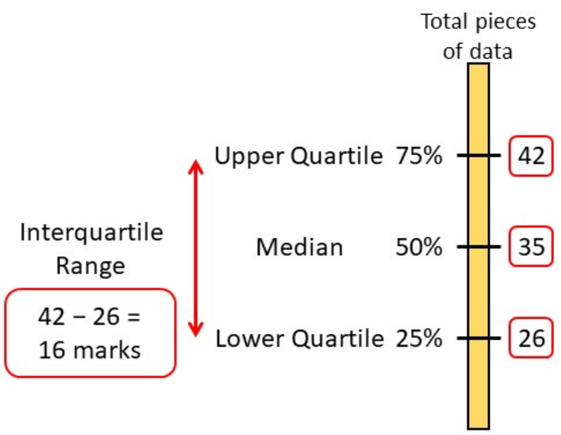
To construct a cumulative frequency graph and read off the Quartiles we do the following:

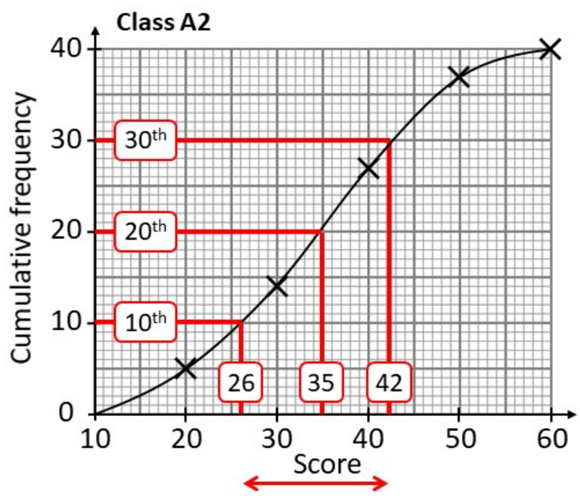
A cumulative frequency graph allows us to **measure the spread** of data.

By finding the value at each **quartile** (quarter) of the data, we can find the **median** and the spread of the **middle 50%.**

This measure eliminates extreme values.

| Score | Students (Frequency) | Cumulative Frequency |
|---|---|---|
| 11-20 | 5 | 5 |
| 21-30 | 9 | 14 |
| 31-40 | 13 | 27 |
| 41-50 | 10 | 37 |
| 51-60 | 3 | 40 |




**Quartiles:**

Lower Quartile, $Q_1$ = $\frac{1}{4}(n+1)^{\text{th}}$ term

Median, $Q_2$ = $\frac{1}{2}(n+1)^{\text{th}}$ term

Upper Quartile, $Q_3$ = $\frac{3}{4}(n+1)^{\text{th}}$ term

Inter Quartile Range = $Q_3 - Q_1$

Semi-Inter Quartile Range = $\frac{Q_3 - Q_1}{2}$

# Section 6 – Algebra

**Basic Algebra rules:**

i. $x + x = 2x$
ii. $x – 2x = x ( 1 – 2) = - x$
iii. $x + y = x + y$

Sign rules

- x - = +

- x + = -

+ x - = -

+ x + = +

**Indices Rules**

i. $x^m . x^n = x^{m+n}$
ii. $x^m \div x^n = x^{m-n}$
iii. $x^0 = 1$
iv. $(x^m)^n = x^{mn}$
v. $x^{-m} = \frac{1}{x^m}$

NB.
An *expression* as no equal sign [=], but an *equation* has an equal sign

**Simplifying:**

When simplifying fractions:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm cb}{bd}$$

Expanding brackets:

$$(a + b)(a + b ) = a^2 + 2ab + b^2$$

**Factorizing**:

1. $ab + ca = a ( b + c)$
2. $x^2y + y^2 x = xy (x + y)$
3. $px^2 + qx + apx + aq = x(px + q) + a (px + q)$
   $= (px + q)(x + a)$
4. $a^2 – b^2 = (a - b)(a + b)$

To factorize a quadratic:

$ax^2 + bx + c$

1. Obtain a, b and c
2. Get two numbers that when multiplied give ac and when added gives b
3. Replace b with those two numbers then factorize

**Solving:**

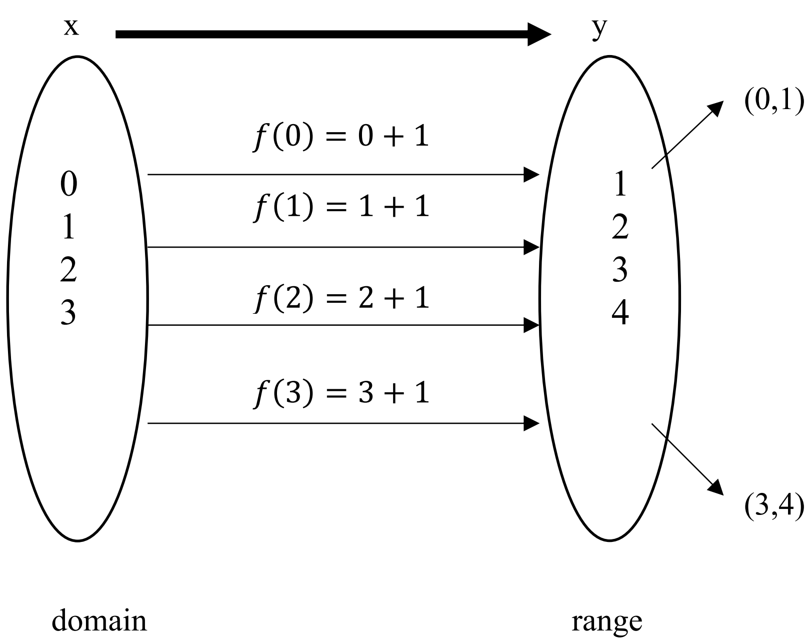
1. $\frac{a}{b} = \frac{c}{d}$ cross multiply to obtain $ad = bc$ then solve for unknown.
2. When solving simultaneously use elimination, if 2 linear equations, or substitution method, if a linear and a quadratic.
3. Direct variation : $a \; \alpha \; b$ and Inverse variation : $a \; \alpha \; \frac{1}{b}$

   $a = kb$ $a = \frac{k}{b}$

## Section 7 – Relations, Functions and Graphs

### Functions

$$f(x) = x + 1$$




$\therefore 1 to 1 mapping$

Eg

If $f(x) = \frac{3x-1}{2}$

$$f(1) = \frac{3(1)-1}{2} = \frac{2}{2} = 1$$

$$f(2) = \frac{3(2)-1}{2} = \frac{6-1}{2} = \frac{5}{2}$$

replace x with 3x

$$f(3x) = \frac{3(3x)-1}{2} = \frac{9x-1}{2}$$

$$f(2x+1) = \frac{3(2x+1)-1}{2}$$

$$= \frac{6x+3-1}{2} = \frac{6x+2}{2}$$

$$= 3x + 1$$

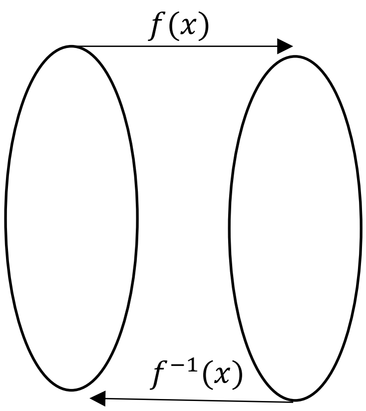
## Composite Functions

functions of functions, substitute one function into the next
eg

$$f(x) = 2x - 1, g(x) = \frac{x}{4}$$
$$f(s(x)) = f\left(\frac{x}{4}\right)$$
$$= 2\left(\frac{x}{4}\right) - 1$$
$$= \frac{x}{2} - 1$$
$$g(f(x)) = g(2x - 1) = \frac{2x - 1}{4}$$

$$f^2(x) = f(f(x))$$
$$f(2x - 1)$$
$$= 2(2x - 1) - 1$$
$$= 4x - 2 - 1$$
$$= 4x - 3$$

## Inverse of a function




Steps
1)let y = f(x)
2) interchange x and y
3) Solve for y
4) $y = f^{-1}(x)$

$$e.g \quad f(x) = 2x - 1$$
$$let y = 2x - 1$$
$$interchange\ x\ and\ y$$
$$x = 2y - 1$$
$$Solve\ for\ y$$
$$2y - 1 = x$$
$$2y = x + 1$$
$$y = \frac{x + 1}{2}$$
$$f^{-1}(x) = \frac{x + 1}{2}$$

Co-ordinate Geometry:

**Distance between two points**: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**Mid-point:** $\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

**Gradient**: m = $\frac{y_2-y_1}{x_2-x_1}$

*parallel lines have equal gradients* $[m_1 = m_2]$

*perpendicular lines,product of gradients equal* $-1$. $[m_1 m_2 = -1]$

**Equation of a line**: $y = mx + c$  m – gradient
c – y-intercept (cuts the y-axis)

To find the equation of a line:
- find gradient of line
- obtain a point on the line
- substitute in $c = y - mx$

NB.
- *Solving equations simultaneously gives the points of intersection of the equations.*

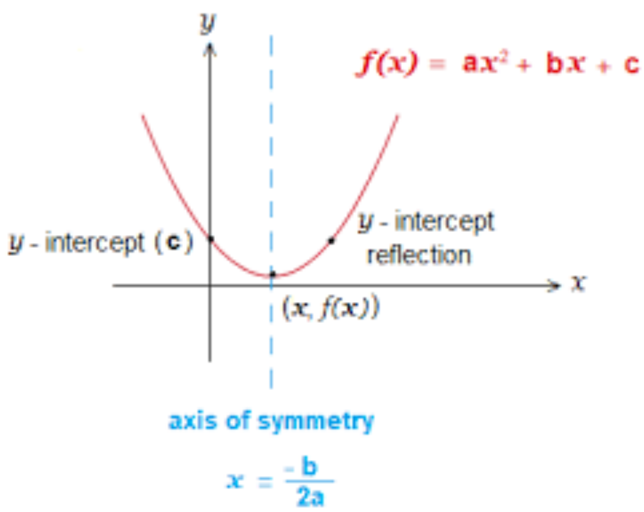
Quadratic:

**General form**: $y = ax^2 + bx + c$ [highest power of x is 2]

**To complete the square**: $y = a(x + h)^2 + k$. where $h = \frac{b}{2a}$ and $k = c - ah^2$

**To sketch a quadratic**:
- Shape: minimum , $a > 0$
  Maximum, $a < 0$

- Turning point : ( - h , k)
- Maximum or minimum value is always k .
- X-value which gives max or minimum value is – h.
- X-intercepts: solve $ax^2 + bx + c = 0$
- Y-intercept: (0,c)




Inequalities:

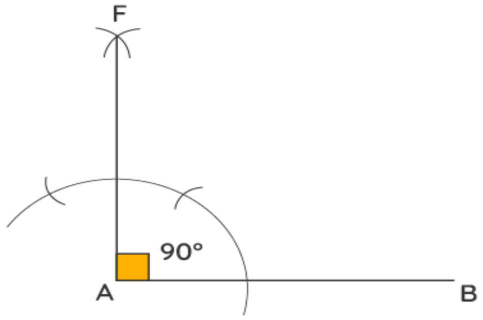
- Solve inequalities like equations, but
- Change the inequality sign when ÷ by a negative
- For < or ≤ : shade below the line
- For > or ≥: shade above the line

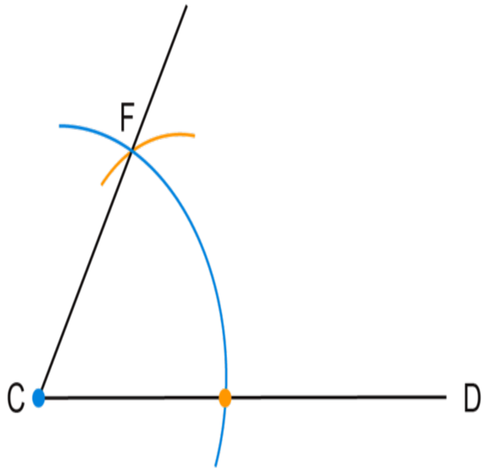
| | |
|---|---|
| < | less than / fewer than |
| > | greater than / more than |
| ≤ | at most / no more than |
| ≥ | at least / no less than |

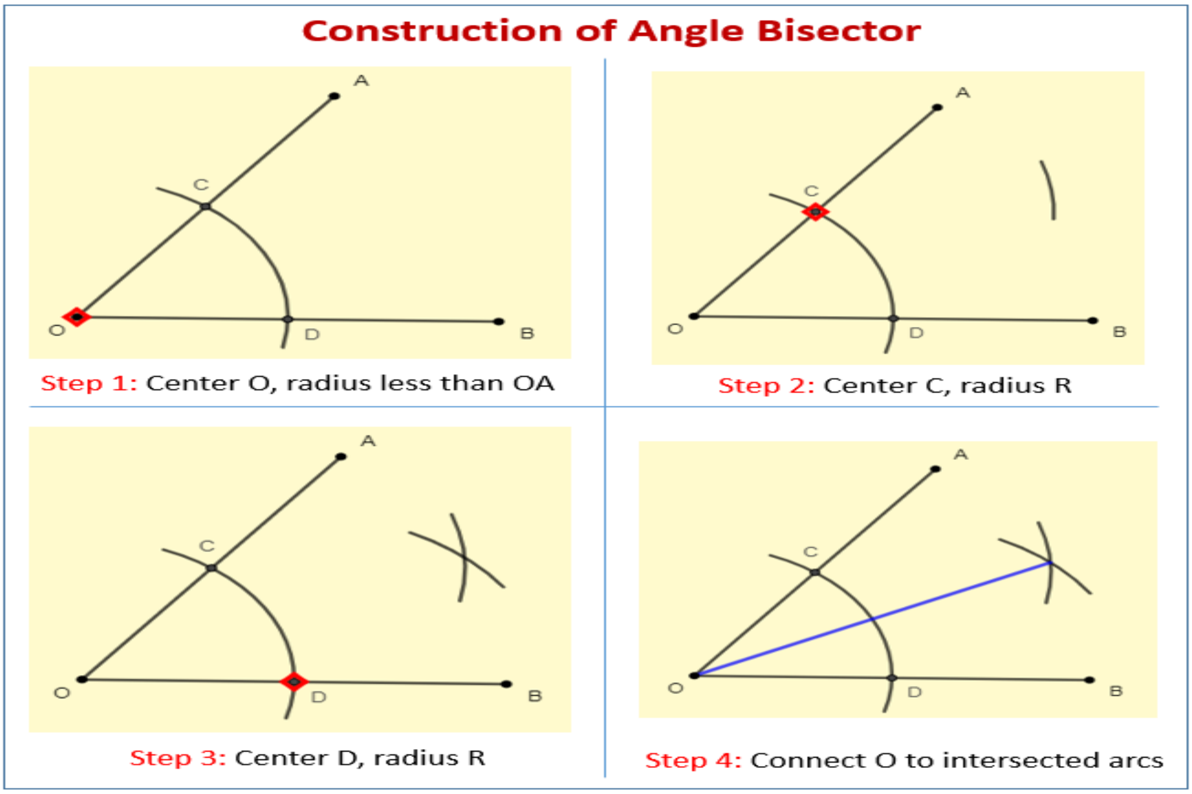
Section 8 – Geometry and Trigonometry

Construction

$90^0$ and $60^0$



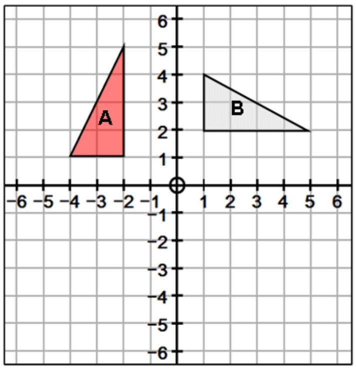



Transformations:

## Describing Rotations

**State…**

1. The centre of rotation
2. The angle of rotation
3. The direction of rotation

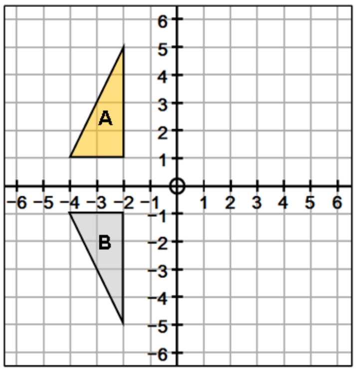



- Centre of rotation is (0,0)
- Angle of rotation is 90°
- Direction of rotation is clockwise

## Describing Reflections

**State…**

1. The line of symmetry



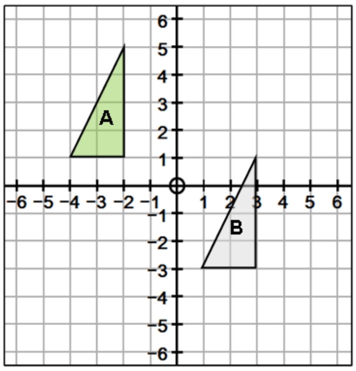

- The line of symmetry is the X axis

## Describing Translations

**State…**

1. Movement left or right
2. Movement up or down

- Or write the column vector

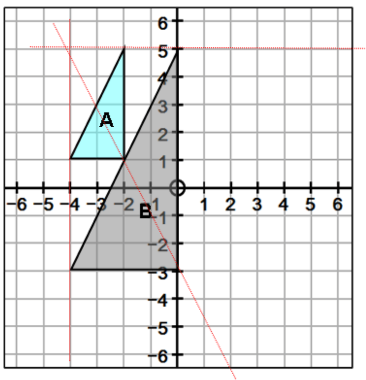



- Translation of 5 to the right and 4 down
- $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

## Describing Enlargements

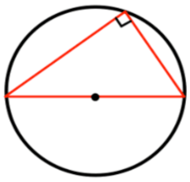
**State…**

1. Centre of enlargement
2. Scale Factor

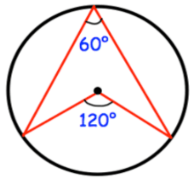



- The centre of enlargement is (-4, 5)
- The scale factor is 2

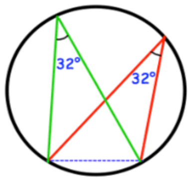
# Circle Theorems



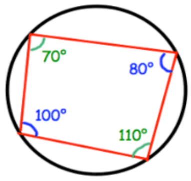
The angle in a semi-circle is 90°



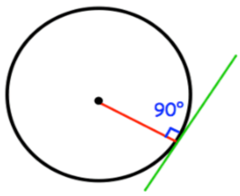

The angle at the circumference is half the angle at the centre



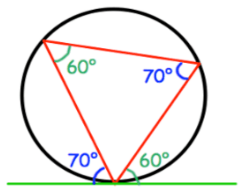

The angles in the same segment from a common chord are equal



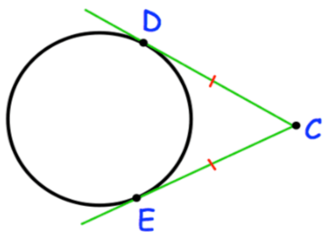

The opposite angles in a cyclic quadrilateral always add to 180°



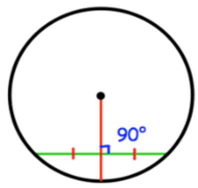

The angle between a radius and a tangent is 90°




**Alternate segment theorem**
The angle between the chord and the tangent is equal to opposite angle inside the triangle.

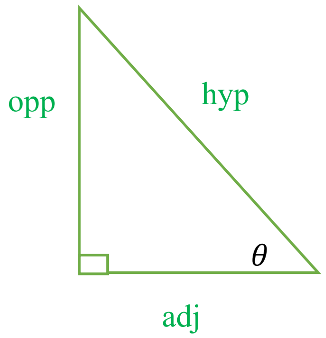



The tangents to a circle from the same point will be equal length




The radius through the midpoint of a chord will bisect the chord at 90°

# Trigonometry




Right – angled

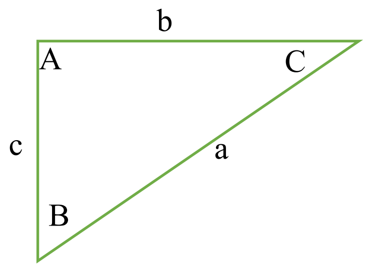
$hyp^2 = opp^2 + adj^2$

$\sin\theta = \frac{0}{4}$

$\cos\theta = \frac{A}{4}$

$\tan\theta = \frac{0}{A}$

$$Area = \frac{1}{2}\, b \times h$$




Cosine rule: **more lengths than angles**

$a^2 = b^2 + c^2 - 2bc\ \cos A$

Sine rule: **more angles than lengths**

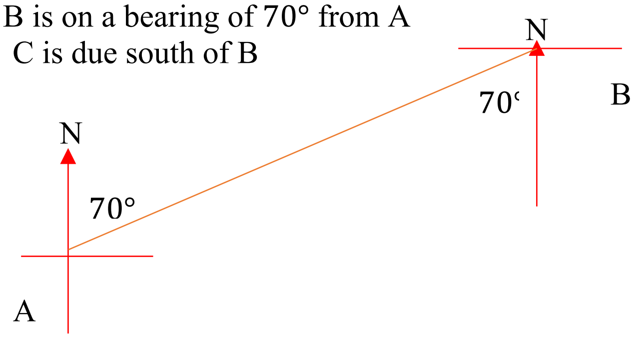
$\frac{a}{\sin A} = \frac{b}{\sin B}$

$Area = \frac{1}{2} ab\ \sin C$

# Bearings

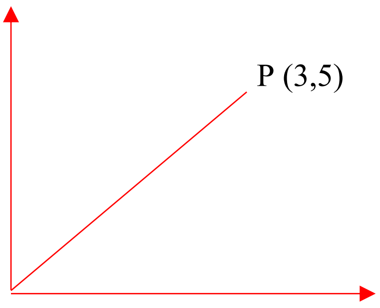
1) Start N
2) Move in a clockwise direction
3) Show all angles

eg. B is on a bearing of 70° from A
C is due south of B



# Section 9 – Vectors and Matrices

Vectors:




$magnitude\ of\ OP$

$|OP| = \sqrt{3^2 + 5^2}$

Position vector, $\overrightarrow{OP} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$\overrightarrow{PO} = -\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$

Addition: $\begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+4 \\ -3+1 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

Subtraction: $\begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 2-4 \\ -3-1 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

## Multiplying Vectors

a) By a scalar

If $\overrightarrow{OP} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$2\,\overrightarrow{OP} = 2\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

b) Two vectors

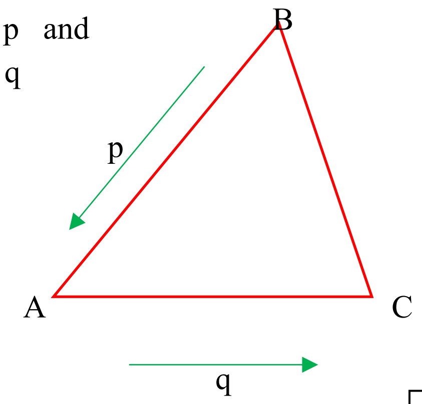
If we have two vectors $\vec{P} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\vec{Q} = \begin{pmatrix} c \\ d \end{pmatrix}$ then

**P.Q = ad + bc**

is called dot or scalar product

## Displacement Vectors

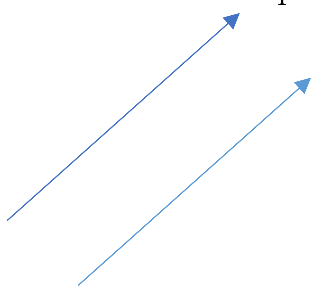
If $\overrightarrow{AB}$ = p and
$\overrightarrow{AC}$ = q



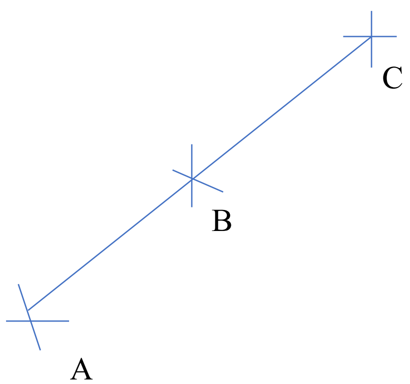

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = -p + q$
alternate route from B to C

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

parallel vector are multiples of each other a=kb



## Collinear




To show collinear
1) show
$\overrightarrow{AB} \parallel \overrightarrow{BC}$
2) Show
$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

# Matrices

A matrix is a set of elements in rows or columns.

Eg. $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ →row 1, →row2, σ

↓ Column 1 ↓ Column 2

Name or size= no. of rows × no. of columns

Give the size:

1) $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ Column Matrix

$3 \times 1$

2) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Square Matrix

$2 \times 2$ NB. Identity Matrix

3) $\begin{pmatrix} 4 & 2 \end{pmatrix}$ Row Matrix

$1 \times 2$

4) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Null Matrix

## Addition and Subtraction

If we have:

$A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ , $B = \begin{pmatrix} 3 & -1 \end{pmatrix}$ , $C = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

To add or subtract:

1) Matrices must be the same size.
2) Add or subtract the element in the same position.

Eg. 1) $A + B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}_{2 \times 1} + \begin{pmatrix} 3 & -1 \end{pmatrix}_{1 \times 2}$

$\therefore$ not same size

We cannot add

2) $A + C = \begin{pmatrix} 1 \\ 2 \end{pmatrix}_{\boxed{2 \times 1}} + \begin{pmatrix} 4 \\ 3 \end{pmatrix}_{\boxed{2 \times 1}} = \begin{pmatrix} 1 & + & 4 \\ 2 & + & 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

## Multiplication

### (a) By a scalar

If we have $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Then $KA = k \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$= \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$

Eg. If $A = \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix}$ then

(a) $2A = 2 \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 4 & -6 \\ 8 & 10 \end{pmatrix}$

(b) $3A = 3 \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 6 & -9 \\ 12 & 15 \end{pmatrix}$

### (c) Two Matrices

To multiply two matrices

(1) No. of columns in first matrix = no. of rows in second matrix

ie. $a \times \boxed{b \;.\; b} \times c$

(2) Product is no. of rows in matrix one by no. of columns in matrix two.

ie. $a \times \boxed{b \;.\; b} \times c$.  Product $= a \times c$

(3) Always rows in matrix one × (by) columns in matrix 2.

(1) If $A = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 3 & -1 \end{pmatrix}$

$2 \times \boxed{2 \quad 2} \times 2 \equiv 2 \times 2$

$$AB = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} (2 \times 10) + (-1)(3) & (2)(1) + (-1)(-1) \\ (3 \times 0) + (2 \times 3) & (3)(1) + (2)(-1) \end{pmatrix}$$

$2 \times 2$

$$= \begin{pmatrix} -3 & 3 \\ 6 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} 0 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (0 \times 2) + (1 \times 3) & (0 \times -1) + (1 \times 2) \\ (3 \times 2) + (-1 \times 3) & (3 \times -1) + (2 \times -1) \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ 3 & -5 \end{pmatrix}$$

## Determinant

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then the determinant, det $A$ or $|A|$

$|A| = ad - bc$

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Example $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$

$|A| = (2)(4) - (-1)(3)$

$= 8 + 3 = 11$

$A = \begin{pmatrix} 2 & 4 \\ -1 & -3 \end{pmatrix}$

$|A| = (2)(-3) - (4)(-1)$

$= -6 + 4$

$= -2$

$B = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

$|B| = (1)(4) - (2)(2)$

$= 4 - 4$

$= 0$

If det in $\neq 0$ then matrix is said to be non-singular.
If det $= 0$, then the matrix is Singular.

If $A = \begin{pmatrix} 2 & P \\ 3 & 1 \end{pmatrix}$ in a Singular matrix, Find p

Since matrix is singular

$$|A| = 0 = (2)(1) - p(3)$$
$$0 = 2 - 3p$$
$$p = \frac{-2}{-3}$$
$$p = \frac{2}{3}$$

If $A = \begin{pmatrix} 2 & 3 \\ p & 4 \end{pmatrix}$ is a singular matrix, Find p.

$$\text{Det } A = 0 = (2)(4) - (3)(p)$$
$$0 = 8 - 3p$$
$$3p = 8$$
$$p = \frac{8}{3}$$

## Inverse of a matrix

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then its inverse, $A^{-1}$ is $A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

NB. A must be non-singular

Example. If $A = \begin{pmatrix} 2 & -3 \\ 1 & 3 \end{pmatrix}$, then det $A = (2)(3) - (-3)(1) = 6 + 3 = 9$

$$\therefore A^{-1} = \frac{1}{9}\begin{pmatrix} 3 & 3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{9} & \frac{3}{9} \\ \frac{-1}{9} & \frac{2}{9} \end{pmatrix}$$

$A = \begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix}$

$$\text{Det } A = -6 - (-4)$$
$$= -6 + 4$$
$$= -2$$

$$A^{-1} = \frac{1}{-2}\begin{pmatrix} -3 & 4 \\ -1 & 2 \end{pmatrix}$$

$B = \begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$

$$|B| = (2)(3) - (6)(1)$$
$$= 6 - 6$$
$$= 0$$

$\therefore$ $B$ is singular

$\therefore$ no inverse

$$\frac{1}{0} = \infty$$

$Ax = b$
(1) Matrix form
(2) $A^{-1}$
(3) $\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} . b$

## Solving Simultaneous Matrix Method

$2x + y = 3$

$3x - 2y = 1$

Write in matrix form

$$\begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$Ax = b$

Where

$A, x$ and $b$ are matrices.

$$= \frac{1}{-7} \begin{pmatrix} -2 & -1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$= \frac{1}{-7} \begin{pmatrix} -6 & + & (-1) \\ -9 & + & 2 \end{pmatrix}$$

$$= \frac{1}{-7} \begin{pmatrix} -7 \\ -7 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-7}{-7} \\ \hline \frac{-7}{-7} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

So $x = A^{-1}.b$

$$A = \begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-7} \begin{pmatrix} -2 & -1 \\ -3 & 2 \end{pmatrix}$$

Since

$x = A^{-1} . b$

$$= \frac{1}{-7} \begin{pmatrix} -2 & -1 \\ -3 & 2 \end{pmatrix} . \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$