C.S.E.C. MATHEMATICS SUMMARY 2024

With Past Paper Questions (Jan 2021 – Jan 2025)

Section 1 – Number Theory and Computation

Sets of numbers:

- Natural numbers, N = {1, 2, 3,….}
- Whole numbers, W = {0, 1, 2, 3, …..}
- Integers, Z = { …, -2, -1, 0, 1, 2, …}
- Rational numbers, Q = { $\frac{p}{q}$, p and q are integers, q ≠ 0}
- Irrational numbers, $\bar{Q}$ = { , $\sqrt{2}$, π, ….}
- Real numbers, R = Q ∪ $\bar{Q}$

Significant figures rules:

1. All non-zero numbers ARE significant.
2. Zeros between two non-zero digits ARE significant.
3. Leading zeros are NOT significant. .e.g. 0.0045 has 2 sig. fig.
4. Trailing zeros to the right of the decimal ARE significant. e.g. 45.00 has 4 sig. fig.
5. Trailing zeros in a whole number with decimal shown ARE significant.
6. Trailing zeros in a whole number with no decimal shown are NOT significant.

Properties of numbers:

a) Closure: If a, b ∈ R then a*b ∈ R.

b) Associative: (x + y) + z = x + (y + z)

c) Commutative: x + y = y + x and x . y = y . x.

d) Distributive: x . (y + z) = x . y. + x . z

e) Additive Identity: x + 0 = 0 + x = x.

f) Multiplicative Identity: x . 1 = 1 . x = x

g) Additive Inverse: x + (-x) = (- x) + x = 0

h) Multiplicative Inverse: x.$\left(\frac{1}{x}\right)$=$\left(\frac{1}{x}\right)$. x =1

Ratios:

A ratio of **a : b : c** implies that the fractions being shared are

$$\frac{a}{a+b+c} : \frac{b}{a+b+c} : \frac{c}{a+b+c}$$

Jan 2021 #1

1. (a) (i) Using a calculator, or otherwise, calculate the EXACT value of

$$1\frac{4}{7} + \frac{2}{3} - 1\frac{5}{6}.$$

(ii) Write the value of $\frac{\sqrt[3]{27}}{9^2}$ as a fraction in its LOWEST terms.

(b) The thickness of one sheet of cardboard is given as 485×10^{-2} mm. A construction worker uses 75 sheets of the cardboard, stacked together, to insulate a wall.

(i) Show that the exact thickness of the insulation is 363.75 mm.

(ii) Write the thickness of the insulation

a) correct to 2 significant figures

b) correct to 1 decimal place

c) in standard form.

(c) Marko is on vacation in the Caribbean. He changes 4500 Mexican pesos (MXN) to Eastern Caribbean dollars (ECD). He receives 630 ECD.

Complete the statement below about the exchange rate.

1 ECD = ... MXN

Jan 2021 #7

7. The diagrams below show a sequence of figures made up of circles with dots. Each figure has one dot at the centre and 4 dots on the circumference of each circle. The radius of the first circle is one unit. The radius of each new circle is one unit greater than the radius of the previous circle. Except for the first figure, a portion of each of the other figures is shaded.

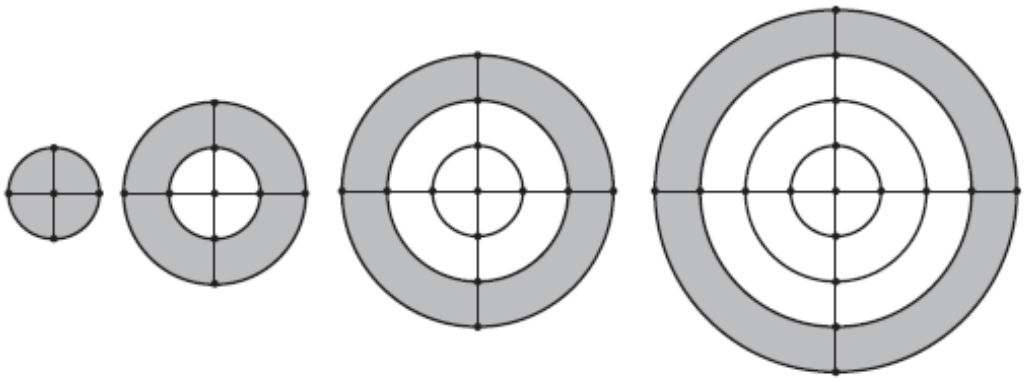


Figure 1 **Figure 2** **Figure 3** **Figure 4**

(a) Complete the rows in the table below for Figure 5 and Figure n.

Figure Number	Number of Dots	Area of Outer (Largest) Circle	Area of Shaded Region	Total Length of Circumference of all Circles
1	5	π	π	2π
2	9	4π	3π	6π
3	13	9π	5π	12π
4	17	16π	7π	20π
(i) 5	_____	25π	_____	_____
⋮	⋮	⋮	⋮	⋮
(ii) n	_____	_____	_____	_____

(3 marks)

(4 marks)

(b) Determine the value of n, when the number of dots in Figure n is 541.

(c) Write down, in terms of p and π, the area of the LARGEST circle in Figure $3p$.

May/June 2021 #1

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

$$1\frac{4}{7} \div \frac{2}{3} - 1\frac{5}{7}.$$

(c) In 1965, the population of Country *A* was 2 714 000. In 2015, the population was 3 663 900.

(i) a) Write the population in 2015 correct to 3 significant figures.

b) Write the population in 1965 in standard form.

(ii) Determine the percentage increase in the population from 1965 to 2015.

(d) The ratio of teachers to male students to female students in a school is 3:17:18. If the TOTAL **number of students** in the school is 630, determine the **number of teachers** in the school.

May/June 2021 #7

7. The first 3 figures in a sequence of shapes, formed by connecting lines of unit length, are shown below.

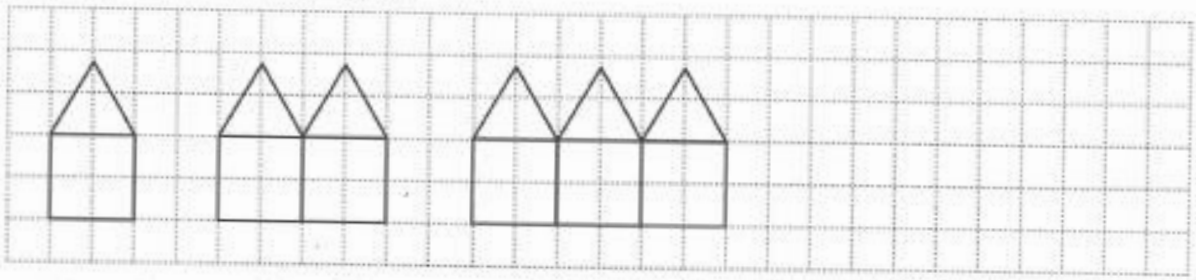


Figure 1 **Figure 2** **Figure 3** **Figure 4**

(a) Draw Figure 4 of the pattern in the space provided above. **(2 marks)**

(b) The number of lines, L, in each shape and the perimeter, P, of the shape follow a pattern. Study the pattern of numbers in each row of the table below and answer the questions that follow.

Complete the table below showing the number of lines and the perimeter of each figure.

Figure	Number of Lines (L)	Perimeter (P)
1	6	5
2	11	8
3	16	11
⋮	⋮	⋮
5	_____	_____
⋮	⋮	⋮
_____	66	_____
⋮	⋮	⋮
n	_____	_____

(i) **(2 marks)**

(ii) **(2 marks)**

(iii) **(2 marks)**

(c) Write a simplified expression, in terms of n, for the **difference**, d, between the number of lines and the perimeter of any figure, n.

Jan 2022 #1

1. (a) Using a calculator, or otherwise, find

(i) the EXACT value of $\dfrac{8.9 + 31.6}{0.75 \times 5.4}$

(ii) the value of $3.9 \tan(18°)$ correct to 1 decimal place.

Jan 2022 #7

7. A sequence of figures is made from lines of unit length and dots. The lines form a series of octagons and squares. The dots are placed at each vertex.

The first 3 figures in the sequence are shown below.

Figure 1

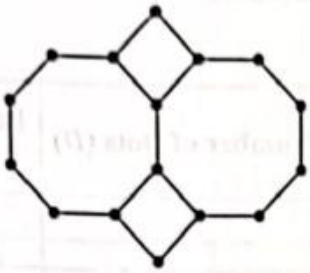


Figure 2

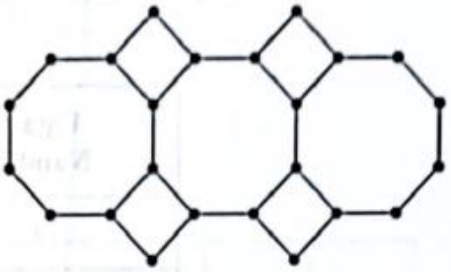
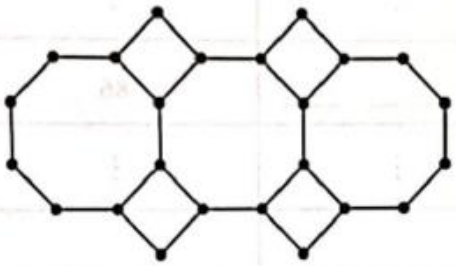


Figure 3

(a) Figure 3 of the sequence is shown by itself below. Add more lines of unit length and dots to Figure 3 to correctly represent Figure 4.



(b) The number of dots, D, and the number of unit lines that form the perimeter of the shape, P, form a pattern. The values for D and P for the first 3 figures are written in the table below. Study the pattern of numbers in each row of the table.

Complete the rows numbered (i), (ii) and (iii).

	Figure Number	Number of Dots (D)	Perimeter of Figure (P)	
	1	8	8	
	2	16	14	
	3	24	20	
(i)	4	_____	_____	**(2 marks)**
	⋮	⋮	⋮	
(ii)	_____	_____	86	**(2 marks)**
	⋮	⋮	⋮	
(iii)	n	_____	_____	**(2 marks)**

(c) For any figure, $n > 1$, the number of dots, D, is greater than its perimeter, P. Determine the value of n for a figure in which the difference between D and P is 36.

May/June 2022 #1

1. (a) Using a calculator, or otherwise, find the

(i) EXACT value of

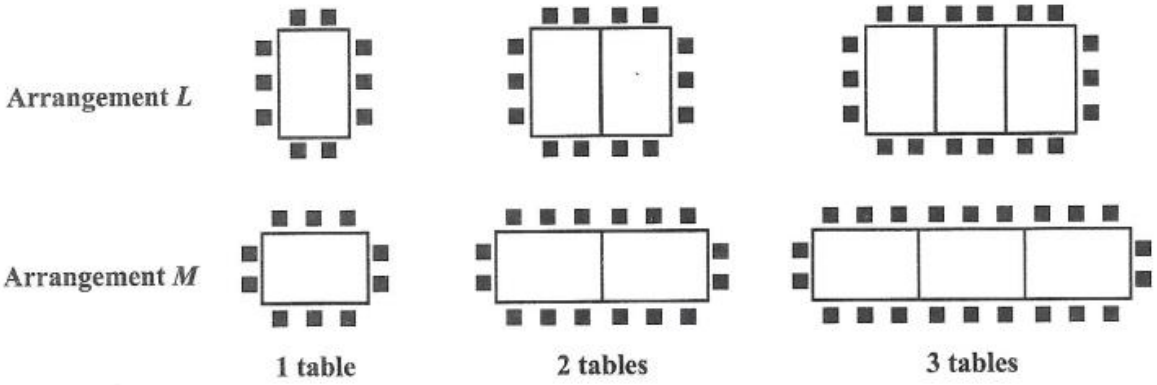
a) $\frac{7}{8} + \frac{1}{6} \div \frac{2}{9}$

b) $\frac{8}{0.4^3}$

(ii) value of $\sqrt{26.8} - 2.5^{\frac{3}{2}}$, correct to 2 decimal places.

May/June 2022 #7

7. At an entertainment hall, tables and chairs can be arranged in two different ways as shown in the diagram below.



(a) Draw the diagram for **4 tables** using **Arrangement *L***.

(b) The number of chairs, C, that can be placed around a given number of tables, T, for either arrangement, L or M, forms a pattern. The values for C for the first 3 diagrams for both arrangements are shown in the table below. Study the pattern of numbers in each row of the table.

Complete the rows numbered (i), (ii) and (iii).

	Number of Tables (T)	Arrangement L	Arrangement M	
		Number of Chairs (C)	Number of Chairs (C)	
	1	10	10	
	2	14	16	
	3	18	22	
(i)	4	_____	_____	**(2 marks)**
	⋮	⋮	⋮	
(ii)	_____	_____	130	**(2 marks)**
	⋮	⋮	⋮	
(iii)	n	_____	_____	**(2 marks)**

(c) Leon needs to arrange tables to seat 70 people for a birthday party. Which of the arrangements, L or M, will allow him to rent the LEAST number of tables?

Use calculations to justify your answer.

Jan 2023 #1

1. (a) (i) By rounding each number in the expression below to **one significant figure**, estimate the value of

$$\frac{\sqrt{108}}{19.72 + 5.296}.$$

(ii) Find the EXACT value of

$$3\frac{3}{8} \div \left(\frac{5}{12} + \frac{1}{3}\right).$$

Give your answer as a mixed number in its **simplest** form.

(b) Due to the COVID-19 pandemic, the number of available seats in a hall was reduced from 125 to 93. Calculate the percentage decrease in the number of available seats.

Jan 2023 #7

7. The grid below shows the first 3 figures in a sequence. Each figure is made using a set of small squares of unit length that are **both** coloured (shaded) and white (unshaded).

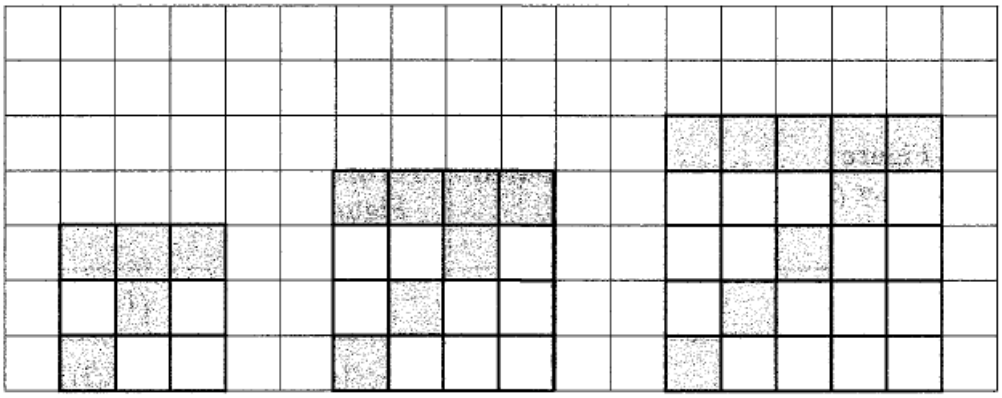


Figure 1 **Figure 2** **Figure 3**

(a) In the space provided below, draw Figure 4 of the sequence.

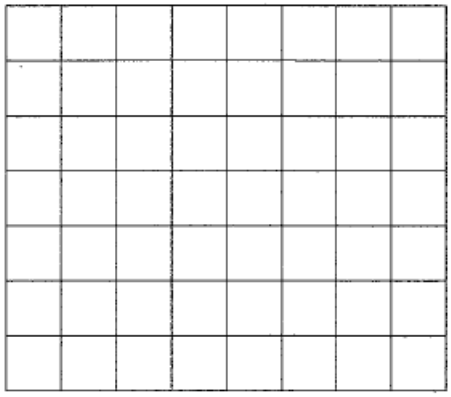


Figure 4

(b) The number of coloured squares, C, the total number of squares, T and the perimeter of the figure, P, follow a pattern. Study the patterns in the table below and answer the questions that follow.

Complete Rows (i), (ii) and (iii) in the table below.

	Figure Number *(F)*	**Number of Coloured Squares** *(C)*	**Perimeter of Figure** *(P)*	**Total Number of Squares** *(T)*	
	1	5	12	$(1+2)^2 = 9$	
	2	7	16	$(2+2)^2 = 16$	
	3	9	20	$(3+2)^2 = 25$	
	⋮	⋮	⋮	⋮	
(i)	11	_____	52	_____	**(2 marks)**
	⋮	⋮	⋮	⋮	
(ii)	_____	49	_____	$(23+2)^2 = 625$	**(2 marks)**
	⋮	⋮	⋮	⋮	
(iii)	n	_____	_____	_____	**(3 marks)**

(c) How many **white squares** are in Figure 11.

May/June 2023 #1

1. (a) Find the EXACT value of

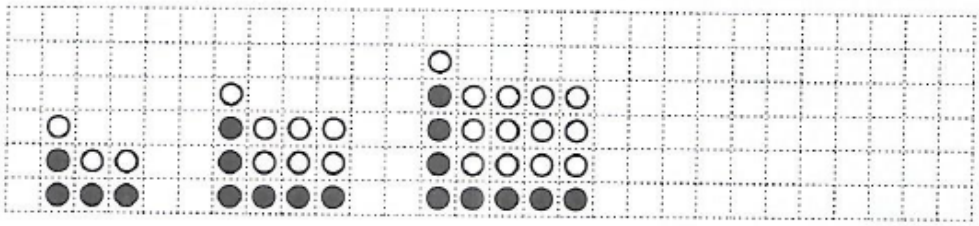
$$\frac{5}{6} + \frac{2}{3} - \frac{12}{35} \times \frac{7}{9}.$$

(b) (i) Calculate the value of $\sqrt{1 - (\cos 37°)^2}$ correct to 3 decimal places.

(ii) Write 0.00527 in standard form.

May/June 2023 #7

7. A sequence of designs is made using black discs and white discs. The first 3 designs in the sequence are shown below.



Design 1 Design 2 Design 3 Design 4

(a) In the space provided on the grid above, draw Design 4. **(2 marks)**

(b) The number of white discs, W, the number of black discs, B, and the total number of discs, T, that form each design follow a pattern. The values for W, B and T for the first 3 designs are shown in the table below. Study the pattern of numbers in the table.

Complete Rows (i), (ii) and (iii) in the table below.

Design Number (P)	Number of White Discs (W)	Number of Black Discs (B)	Total Number of Discs (T)	
1	$(1 \times 1) + 1 + 1 = 3$	4	7	
2	$(2 \times 2) + 2 + 1 = 7$	6	13	
3	$(3 \times 3) + 3 + 1 = 13$	8	21	
⋮	⋮	⋮	⋮	
(i) 9	(..... × .....) + ... + ... = ...	.....	111	**(2 marks)**
⋮	⋮	⋮	⋮	
(ii)	$(20 \times 20) + 20 + 1 = 421$	.....	.....	**(2 marks)**
⋮	⋮	⋮	⋮	
(iii) n	.....	.....	.....	**(3 marks)**

(c) Stephen has 28 black discs and 154 white discs, and wants to make Design 12. Explain why it is NOT possible for him to make Design 12.

Jan 2024 #1

1. (a) Express, as a single fraction in its **simplest** form,

$$1 - \left(\frac{1}{30} + \frac{4}{15} \right).$$

(b) A two-storey car park has a total of 1 020 parking spaces. At 06:30 hours one morning, $\frac{1}{30}$ of the 1 020 spaces are filled. During the next hour, no cars left the car park but another $\frac{4}{15}$ of the 1 020 spaces are filled. Determine the number of parking spaces that are NOT filled at 07:30 hours.

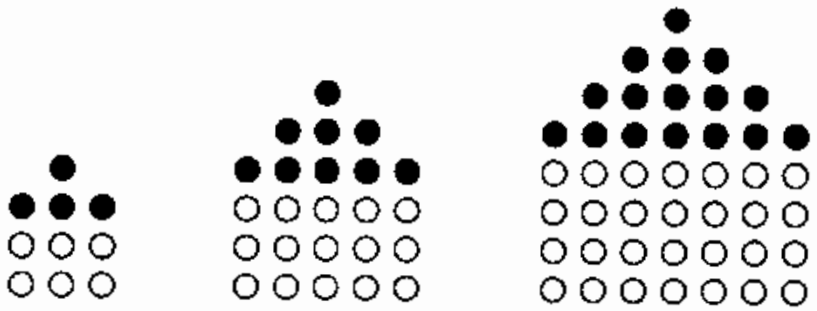
(c) Of the 1 020 parking spaces, 20% are on the top level. How many parking spaces are on the top level?

(d) Some of the spaces are reserved for monthly paying customers. The ratio of reserved spaces to non-reserved spaces is 5:7.

Calculate the number of **non-reserved** parking spaces.

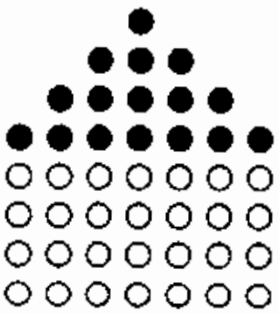
Jan 2024 #7

7. The diagram below shows the first 3 shapes in a sequence, which forms a pattern. Each shape is made using a set of small white counters and black counters.



Shape 1 **Shape 2** **Shape 3**

(a) Complete the diagram below to represent Shape 4.



(b) The number of white counters, W, the number of black counters, B, and the total number of counters, T, that form each shape follow a pattern. The values for W, B and T for the first 3 shapes are shown in the table below. Study the pattern of numbers in each row of the table and answer the questions that follow.

Complete the rows marked (i), (ii) and (iii) in the table below.

	Shape Number *(S)*	**Number of White Counters** *(W)*	**Number of Black Counters** *(B)*	**Total Number of Counters** *(T)*	
	1	$(1 + 1)[2(1) + 1] = 6$	$(1 + 1)^2 = 4$	10	
	2	$(2 + 1)[2(2) + 1] = 15$	$(2 + 1)^2 = 9$	24	
	3	$(3 + 1)[2(3) + 1] = 28$	$(3 + 1)^2 = 16$	44	
(i)	4	**45**	_____	_____	**(2 marks)**
	⋮	⋮	⋮	⋮	
(ii)	_____	_____	**144**	**420**	**(2 marks)**
	⋮	⋮	⋮	⋮	
(iii)	n	(__ + __)[__ + __]	(__ + __)^2	$3n^2 + 5n + 2$	**(2 marks)**

(c) The expression for the total number of counters, $T = W + B$, in Shape S is given by $T = aS^2 + bS + 2$, where a and b are both positive integers.

By substituting suitable values for S, show that the total number of counters in Shape 1 and Shape 3, in terms of a and b, is represented by the equations

$$a + b = 8 \text{ and}$$
$$3a + b = 14 \text{ respectively.}$$

May/June 2024 #1`

1. (a) (i) Calculate the value of $\sqrt{(7.1)^2 + (2.9)^2}$, giving your answer correct to

a) 2 significant figures

b) 2 decimal places.

(ii) Write the following quantities in ascending order.

$\frac{12}{25}$, 0.46, 47%

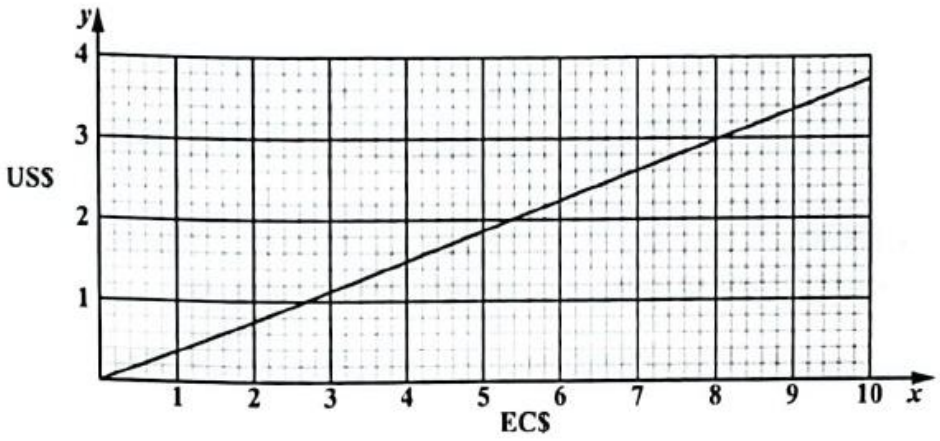
(b) Mahendra and Jaya shared $7 224 in the ratio 7:5. How much MORE money does Mahendra receive than Jaya?

(c) The present population of Portmouth is 550 000. It is expected that this population will increase by 42% by the year 2030.

(i) Write the number 550 000 in standard form.

(ii) Calculate the expected population of Portmouth in 2030.

(d) The graph below can be used to convert between United States dollars (US$) and Eastern Caribbean dollars (EC$).



Using the graph,

(i) convert US$2 to EC$.

(ii) convert EC$70 to US$.

May/June 2024 #7

7. The diagram below shows the first four diagrams in a sequence of regular hexagons. Each regular hexagon is made using sticks of unit length.

Diagram 1 **Diagram 2** **Diagram 3** **Diagram 4**

(a) Complete the diagram below to represent Diagram 5 in the sequence of regular hexagons.

(b) The number of regular hexagons, H, the number of sticks, S, and the perimeter of each figure, P, follow a pattern. The values for H, S and P, for the first 4 diagrams are shown in the table below. Study the pattern of numbers in each row of the table and answer the questions that follow.

Complete the rows marked (i), (ii) and (iii) in the table below.

Diagram Number (D)	Number of Hexagons (H)	Number of Sticks (S)	Perimeter (P)	
1	3	15	12	
2	5	23	16	
3	7	31	20	
4	9	39	24	
(i) 5	.....	.....	28	(2 marks)
⋮	⋮	⋮	⋮	
(ii)	47	191	.....	(2 marks)
⋮	⋮	⋮	⋮	
(iii) n	.....	.....	.....	(3 marks)

(c) Skyla says that she can make one of these figures with a perimeter of EXACTLY 1 005. Explain why she is **incorrect**.

Jan 2025 #1

1. (a) Using a calculator or otherwise, determine the value of

(i) $\frac{2}{3}$ of $\left(\frac{1}{8}+\frac{5}{12}\div\frac{1}{9}\right)$, giving your answer in EXACT form

(ii) $314.2-\frac{26082}{52164}$, giving your answer in standard form.

(b) Jim packed several cases of fruit juice for sale. Each case contained 24 boxes of juice in 3 different varieties, apple, orange and pineapple, in the ratio 2:5:1 respectively.

(i) How many boxes of **pineapple** juice were packed in each case?

Jan 2025 #7

7. A sequence of patterns is made of dots and lines of unit length. Some of these lines form squares. The first three diagrams in the sequence are shown below.

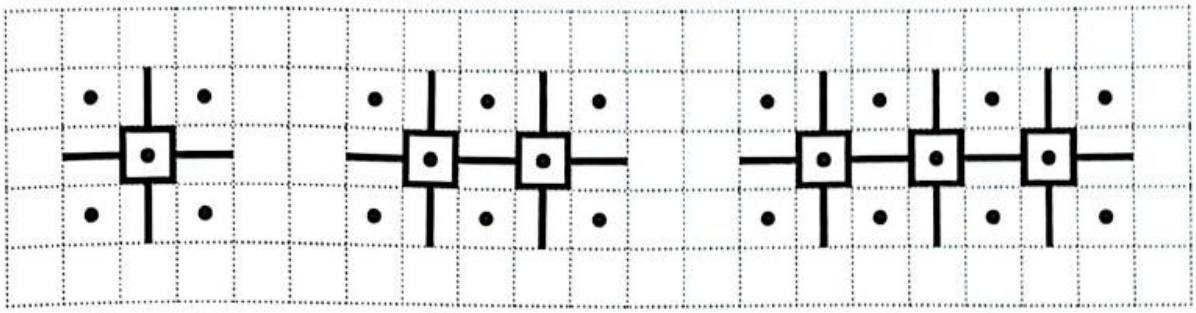


Diagram 1 **Diagram 2** **Diagram 3**

(a) Add more lines and dots to the diagram below to show Diagram 4 of the sequence.

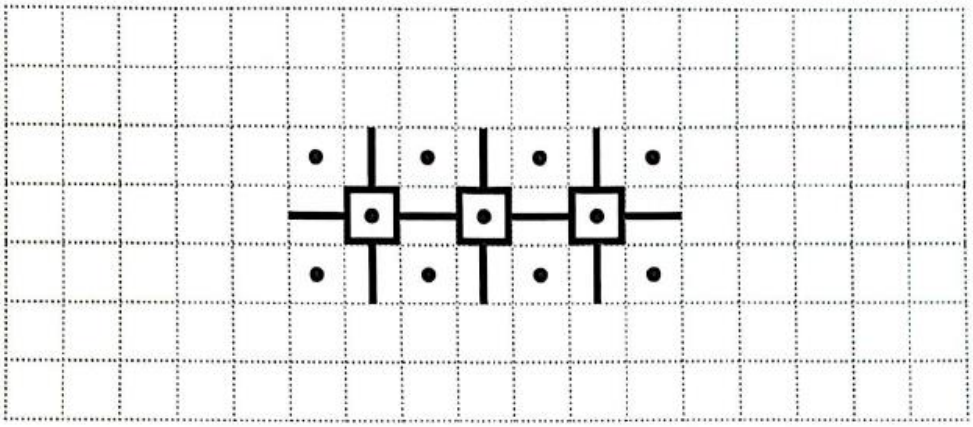


Diagram 4

(b) The number of dots, D, and the number of unit lines that form each diagram, L, form a pattern. The values for D and L for the first 3 diagrams are written in the table below. Study the pattern of numbers in each row of the table.

Complete the rows numbered (i), (ii) and (iii).

	Diagram	Number of Dots (D)	Number of Lines (L)	
	1	5	8	
	2	8	15	
	3	11	22	
(i)	4	_____	_____	**(2 marks)**
	⋮	⋮	⋮	
(ii)	_____	59	_____	**(2 marks)**
	⋮	⋮	⋮	
(iii)	n	_____	_____	**(2 marks)**

149

(c) One of the diagrams in the sequence has 148 **lines**. Calculate the number of **dots** in this diagram.

Section 2 – Consumer Arithmetic

Discount = Selling Price x discount %

Sales Tax = $\frac{total\ sale\ tax}{Selling\ price} \times 100\%$

Profit = Selling Price – Cost price

Loss = Cost price – Selling price

Profit % = $\frac{SP-CP}{CP}$ x 100 %

Simple Interest = $\frac{Principal \times Rate \times time}{100}$

Compound Interest Formula, $A = P(1+i)^n$

A – final amount including principal

P – principal amount

i – interest rate per year

n – number of years

The Compound Interest formula can also be used to determine appreciation and depreciation.

May/June 2021 #1

(b) When Meghan started working, she was paid \$85 each week. After a six-month probationary period, her pay was increased by 20%. How much was she paid each week **after** the increase?

Jan 2022 #1

(b) (i) Ria is paid at the rate of \$13.50 per hour. During a certain week she worked 40 hours. How much did she earn that week?

(ii) Ria worked 4 weeks in the month of August and her gross earnings was \$2 463.75. Her regular week comprised 40 hours and overtime was paid at $1\frac{1}{2}$ times the hourly rate.

Show that Ria worked 15 hours overtime in August.

(iii) In August, 20% of Ria's gross earnings was deducted as tax. How much money does she have left after the deduction?

(iv) Ria invested \$219 of her earnings for 3 years at a rate of 4.5% per annum simple interest. How much interest does she receive after 3 years?

May/June 2022 #1

(b) Children go to a Rodeo camp during the Easter holiday. Ms Rekha buys bananas and oranges for the children at the camp.

(i) Bananas cost \$3.85 per kilogram. Ms Rekha buys 25 kg of bananas and receives a discount of 12%. How much money does she spend on bananas?

(ii) Ms Rekha spends \$165.31, inclusive of a sales tax of 15%, on oranges. Calculate the original price of the oranges.

(iii) The ratio of the number of bananas to the number of oranges is 2:3. Furthermore, there are 24 more oranges than bananas.

Calculate the number of bananas Ms Rekha bought.

Jan 2023 #1

(c) Mica invests a certain amount of money in a bank that pays compound interest at a rate of 2.5% per annum. At the end of 2 years, the value of her investment is \$7 564.50.

Calculate the amount Mica invests.

$$\left(\text{Compound interest}: A = P\left(1+\frac{r}{100}\right)^n, \text{where, } A = \text{total amount after } n \text{ years;}\right.$$
$$P = \text{principal or original value;}$$
$$\left.r = \text{rate of interest per annum; } n = \text{number of years the money is invested}\right)$$

May/June 2023 #1

(c) Haresh works at a call centre for 35 hours each week. He is paid an hourly rate of \$11.20.

(i) Calculate the amount of money Haresh earns in a **four-week month**.

(ii) In a certain week, Haresh works 8 hours overtime. Overtime hours are paid at $1\frac{1}{2}$ times the usual rate of \$11.20 per hour.

Find the TOTAL amount of money Haresh is paid for **that week**.

Jan 2024 #1

(e) The cost for parking at the car park is shown in the table below.

Length of Visit	Cost (\$)
Under 30 minutes	Free
More than 30 minutes and up to 2 hours	\$ 2.25
More than 2 hours and up to 4 hours	\$ 5.50
More than 4 hours and up to 8 hours	\$ 9.25
More than 8 hours and up to 24 hours	\$15.00
One-week ticket	\$40.00

(i) Mikayla leaves the car park at 18:30 hours and pays \$9.25. Determine the **earliest** time she could have arrived at the car park.

(ii) Dhanraj bought a weekly parking ticket for \$40. That week, he visited the car park five **different** times. The length of time he parked his car on each occasion is given below.

25 minutes $\quad 7\frac{1}{2}$ hours $\quad$ 11 hours $\quad 8\frac{3}{4}$ hours $\quad$ 8 hours

Show that Dhanraj saved \$8.50 by buying the weekly ticket.

Jan 2025 #1

(b) Jim packed several cases of fruit juice for sale. Each case contained 24 boxes of juice in 3 different varieties, apple, orange and pineapple, in the ratio 2:5:1 respectively.

(i) How many boxes of **pineapple** juice were packed in each case?

(ii) The profit gained from selling ALL of the boxes of pineapple juice is \$35.64. Each box of pineapple juice was sold at \$3.34.

a) Show that the cost price of a box of pineapple juice is \$2.35.

b) Calculate the percentage profit made on the sale of the boxes of pineapple juice.

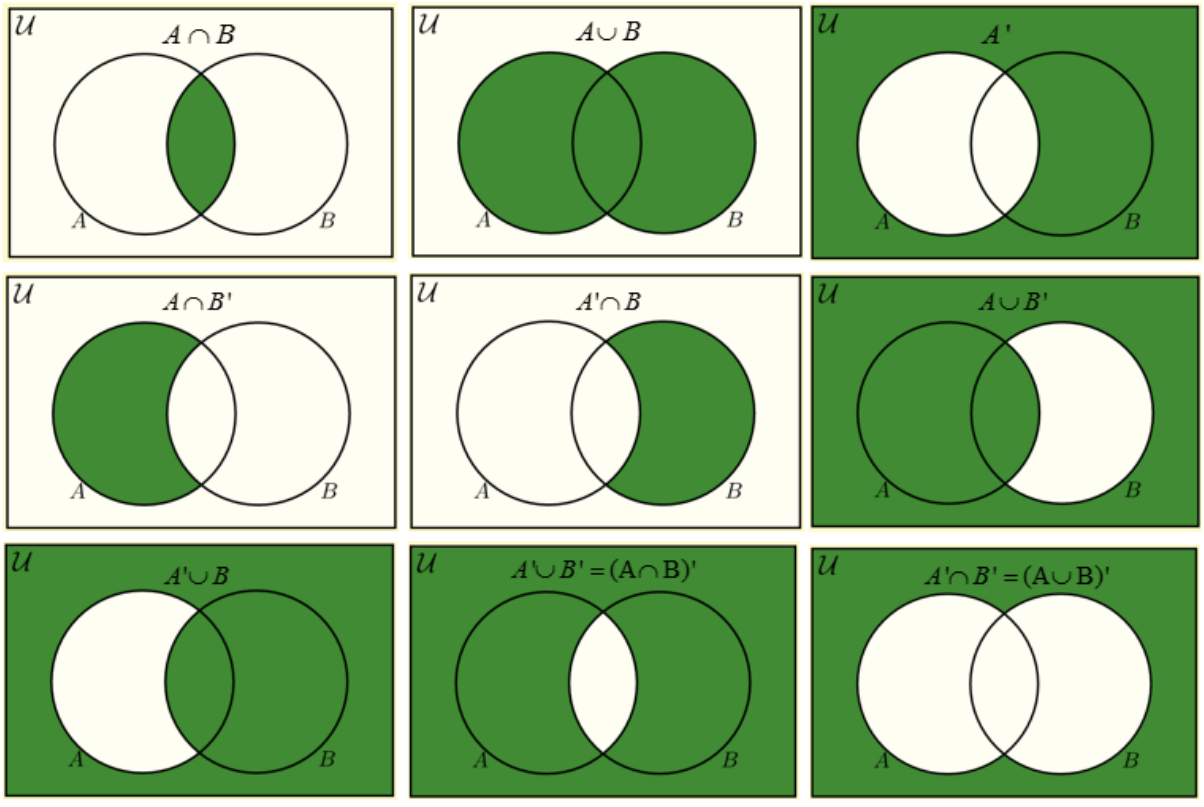
Section 3 – Sets

Set Theory Symbols:

Symbol	Name	Example	Explanation
{ }	Set	$A = \{1,3\}$ $B = \{2,3,9\}$ $C = \{3,9\}$	Collection of objects
$\cap$	Intersect	$A \cap B = \{3\}$	Belong to both set A and set B
$\cup$	Union	$A \cup B = \{1,2,3,9\}$	Belong to set A or set B
$\subset$	Proper Subset	$\{1\} \subset A$ $C \subset B$	A set that is contained in another set
$\subseteq$	Subset	$\{1\} \subseteq A$ $\{1,3\} \subseteq A$	A set that is contained in or equal to another set
$\not\subset$	Not a Proper Subset	$\{1.3\} \not\subset A$	A set that is not contained in another set
$\supset$	Superset	$B \supset C$	Set B includes set C
$\in$	Is a member	$3 \in A$	3 is an element in set A
$\notin$	Is not a member	$4 \notin A$	4 is not an element in set A

N.B. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Venn Diagrams:

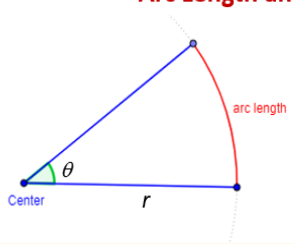


Section 4 – Measurements

Length	Mass
10 mm = 1 cm	1g = 1000mg
100 cm = 1 m	1kg = 1000g
1000 mm = 1 m	1kg = 2.2lbs
1000 m = 1 km	1lb = 16 ounces

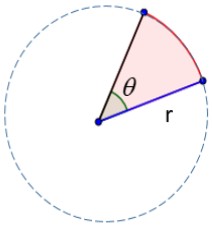
Shape	Volume Formula	Surface Area Formula
Cube	s^{-1} Where s = length of the side	$6s^2$ Where s = Length of the side
Cuboid	lwh Where l = length, w = width and h = height	$2(lw + lh + wh)$ Where l = length, w = width and h = height
Prism	$Area\ of\ base \times height$	$(2 \times area\ of\ base)$ $+$ $(perimeter\ of\ base \times height)$
Cylinder	$\pi r^2 h$ Where r = radius ,h = height	$2\pi r(r + h)$ Where r = radius ,h = height
Hollow Cylinder	$\pi h(R^2 - r^2)$ Where, R = radius of the outer surface, r = radius of the inner surface	$2\pi rh + 2\pi Rh + 2(\pi R^2 - \pi r^2)$ Where, R = radius of the outer surface, r = radius of the inner surface
Cone	$\frac{1}{3}\pi r^2 h$ Where r = radius, h = height	$\pi r\ (r + s)$ Where r = radius, s = slant height
Pyramid	$\frac{1}{3}Bh$ Where B = area of base, h = height	$Any\ pyramid\ = area\ of\ base$ $+area\ of\ each\ of\ the\ lateral\ faces$ OR $Regular\ pyramid$ $= area\ of\ base + \frac{1}{2}\ ps$ Where p = perimeter of the base, s = slant height OR $Square\ pyramid = b^2 + 2bs$ Where b = length of the base, s = slant height
Sphere	$\frac{4}{3}\pi r^3$ Where r = radius	$4\pi r^2$ Where r = radius
Hemisphere	$\frac{2}{3}\pi r^3$ Where r = radius	$3\pi r^2$ Where r = radius

Arc Length and Area of Sector



If θ is measured in degrees then

$$\text{arc length} = \frac{\theta}{360^\circ} \times 2\pi r$$



If θ is measured in degrees then

$$\text{area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$\text{Speed} = \frac{distnace}{time}$$

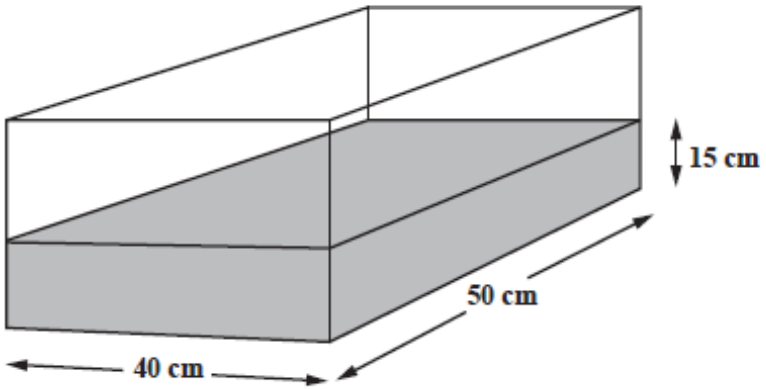
Units: ms^{-1} or kmh^{-1}

Distance = speed x time

Jan 2021 #6

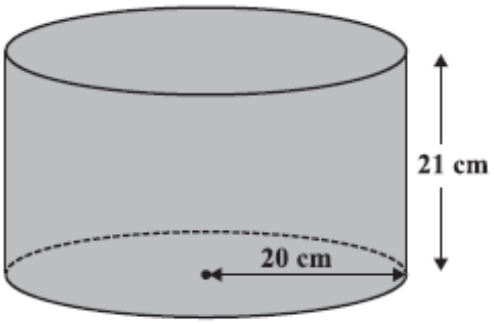
6. In this question, take π to be $\frac{22}{7}$.

The diagram below shows a rectangular tank, with base 50 cm by 40 cm, that is used to store water. The tank is filled with water to a depth of 15 cm.



(a) Calculate the volume of water in the tank.

(b) The cylindrical container shown in the diagram below is used to fetch **more** water to fill the rectangular tank. The container, which is completely filled with water, has a radius of 20 cm and a height of 21 cm.



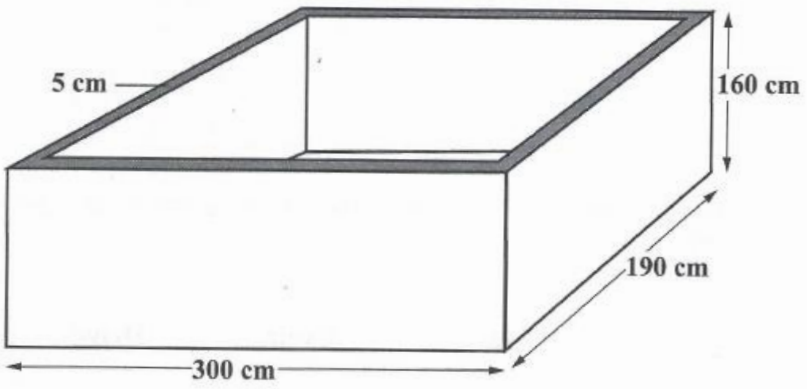
All the water in this container is added to the water in the rectangular tank. Calculate the TOTAL volume of water that is now in the rectangular tank.

(c) Show that the **new** depth of water in the rectangular tank is 28.2 cm.

(d) The vertical height of the rectangular tank is 48 cm. Determine how many more cylindrical containers of water must be poured into the rectangular tank for it to be completely filled.

May/June 2021 #6

6. Farmer Brown makes troughs to feed his farm animals, using wood that is 5 cm thick. As shown in the diagram below, the troughs are rectangular-based, open at the top and have external dimensions of 300 cm by 190 cm by 160 cm.



(a) Show, by calculation, that the internal capacity (volume) of the trough is 8 091 000 cm^3.

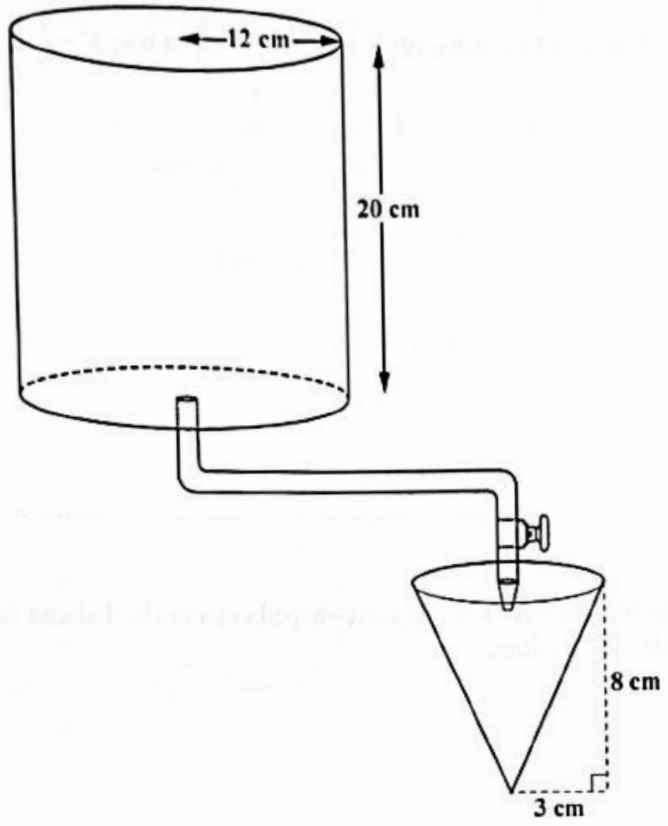
(b) Calculate the volume of wood needed to make a trough.

(c) Farmer Brown must paint the INTERNAL surface of the trough. Given that 1 gallon of paint covers approximately 280 000 cm^2 of surface, determine the TOTAL amount of paint, **in litres**, that is needed to paint the internal surface of the trough.

(1 gallon ≈ 3.79 litres)

Jan 2022 #6

6. At a track meet, a cylindrical container, fitted with a pipe as shown in the diagram below, is used to serve water to athletes. The cylindrical container of radius 12 cm and height 20 cm is **completely** filled with water and the pipe fitted at the bottom dispenses water into cone-shaped cups. The cone-shaped cups have a radius of 3 cm and a height of 8 cm.



(a) Calculate the volume of water in the cylindrical container, **in litres**. Write your answer correct to 2 decimal places. [1 000 cm^3 = 1 litre]

(b) Water flows from the cylindrical container along the pipe into the cone-shaped cups at a rate of 7.8 *ml* per second.

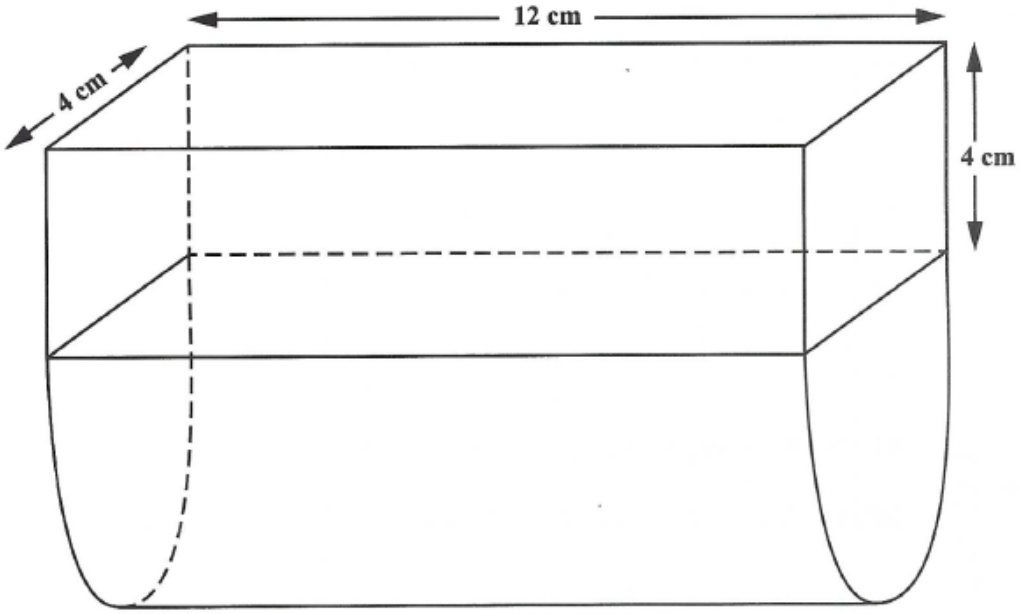
Calculate the time taken to fill ONE of the empty cone-shaped cups. Give your answer correct to the nearest second.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

(c) Determine the number of empty cone-shaped cups in (b) that can be **completely** filled from the cylindrical container.

May/June 2022 #6

6. The diagram below shows a solid made from a semi-circular cylindrical base, with a rectangular prism above it. The diameter of the cylindrical base and the width of the rectangular prism are 4 cm each.



(a) Calculate the TOTAL surface area of the solid.

[The surface area, A, of a cylinder with radius r is $A = 2\pi r^2 + 2\pi rh$].

(b) Calculate the volume of the solid.

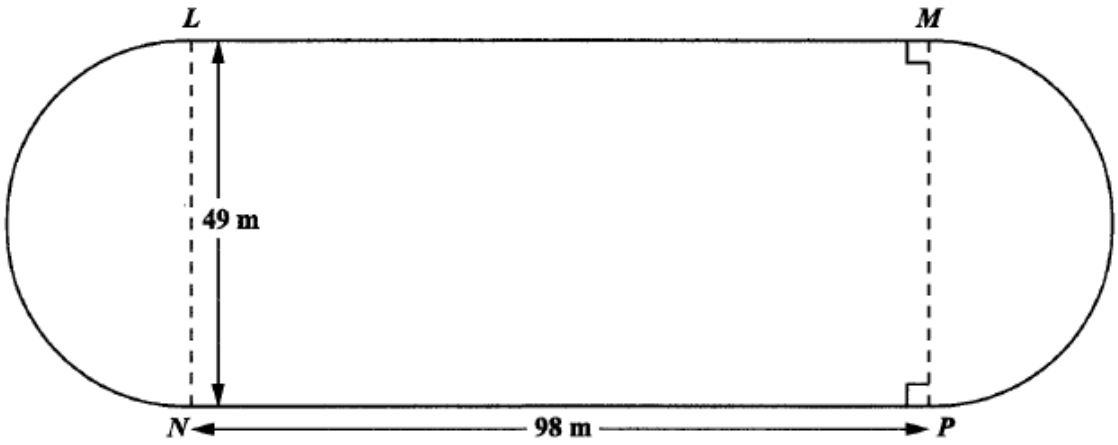
(c) The solid is made from gold. One **cubic centimetre** of gold has a mass of 19.3 grams. The cost of 1 gram of gold is \$42.62.

Calculate the cost of the gold used to make the solid.

Jan 2023 #6

6. In this question, use $\pi = \frac{22}{7}$.

The diagram below shows a scaled drawing of a running track. It consists of a rectangle and two semicircles with diameters LN and MP. $LN = MP = 49$ m and $LM = NP = 98$ m.



(a) (i) Show that the TOTAL length of the running track is 350 m.

(ii) Nathan walks at a constant rate of 1.4 m/s. Calculate the time it will take him to walk 7 laps around the track.

(b) Tafari runs one lap of the track in 68 seconds.

(i) Determine the number of laps Tafari can complete in one hour, running at the same speed.

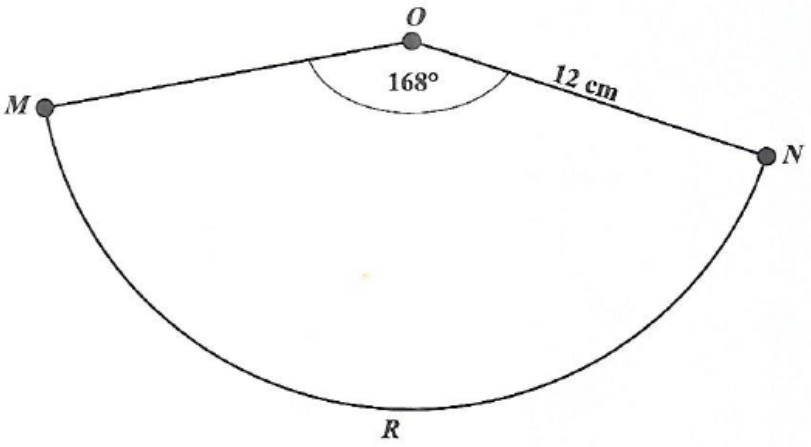
(ii) Nathan completes running one lap of the track every 72 seconds. Tafari and Nathan start running at the same time from point L on the track. Each completed a number of laps of the track. Calculate the LEAST number of laps that each will complete before they are both at point L again at the same time.

Tafari completes ..... laps and Nathan completes ..... laps.

May/June 2023 #6

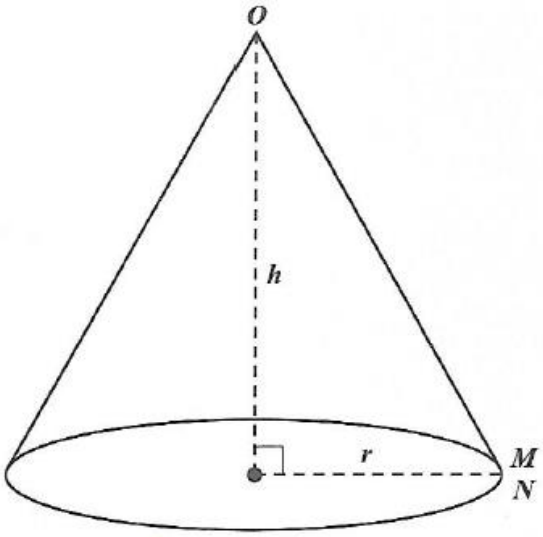
6. [In this question, take $\pi = \frac{22}{7}$ and the volume, V, of a cone with radius r and height h as $V = \frac{1}{3}\pi r^2 h$.]

The diagram below shows a sector $OMRN$, of a circle with centre O, radius 12 cm and sector angle 168°, which was formed using a thin sheet of metal.



(a) Calculate the perimeter of the sector above, made from the thin sheet of metal.

(b) A cone is made from the sector in (a) by joining OM to ON, as shown below.



(i) Calculate the

a) radius, r, of the cone

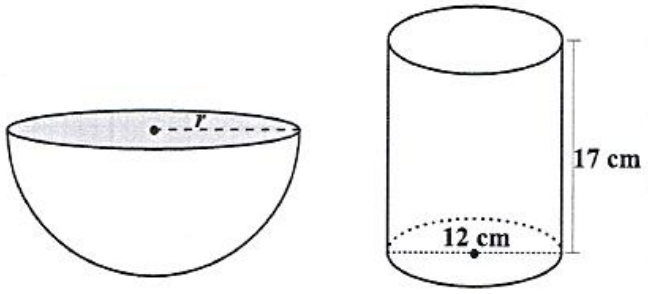
b) height, h, of the cone.

(ii) Calculate the capacity of the cone, **in litres**.

Jan 2024 #6

6. [In this question, use $\pi = \frac{22}{7}$.]

The diagrams below show a hemispherical bowl and a cylindrical tin. The diameter of the cylindrical tin is 12 cm, the height is 17 cm and the radius of the bowl is r.

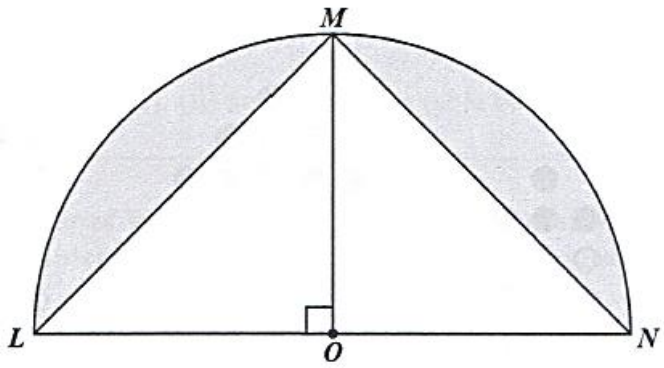


(a) (i) Show that the volume of the cylindrical tin is 1 923 cm^3, correct to 4 significant figures.

(ii) The bowl is completely filled with soup. When all the soup is poured into the empty cylindrical tin, 90% of the volume of the tin is filled. Calculate the radius of the bowl.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

(b) In the diagram below, points L, M and N are on the circumference of a semicircle, with centre O, and a radius of 18 cm.

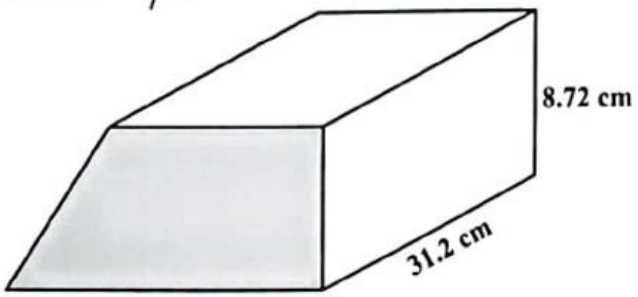


Calculate the TOTAL area of the shaded sections in the diagram.

May/June 2024 #6

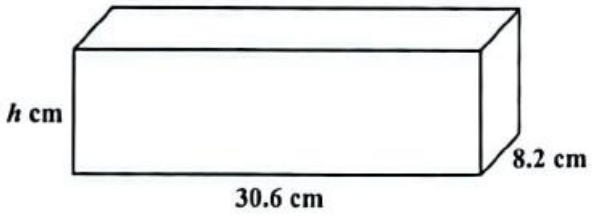
6. (a) The diagram below shows a gold bar in the shape of a trapezoidal prism. Its volume is 2 886 cm³. The length and height of the prism are indicated on the diagram.

[In this question, take $\pi = \frac{22}{7}$.]



(i) Calculate the area of the **shaded** cross-section of the trapezoidal prism.

(ii) The cuboid-shaped gold bar shown below has the same volume as the trapezoidal prism-shaped gold bar displayed at (a).



Calculate the height, h, of the cuboid-shaped gold bar.

(b) The trapezoidal gold bar is melted down and **all** the gold is used to make SIX identical spheres.

Calculate, for EACH sphere of gold, its

(i) radius

$\left[\text{The volume, } V, \text{ of a sphere with radius, } r \text{ is } V = \frac{4}{3}\pi r^3.\right]$

(ii) surface area

[The surface area, A, of a sphere with radius r, $A = 4\pi r^2$.]

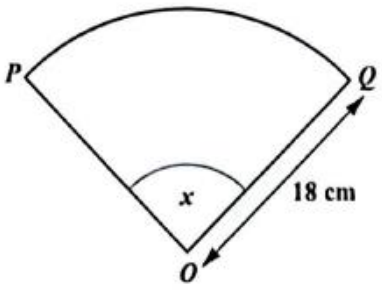
(iii) mass, **to the nearest kilogram**, given that the density of gold is 19.3 g/cm³.

$\left[\text{Density} = \frac{\text{mass}}{\text{volume}}\right]$

Jan 2025 #6

6. [In this question, use $\pi = \frac{22}{7}$.]

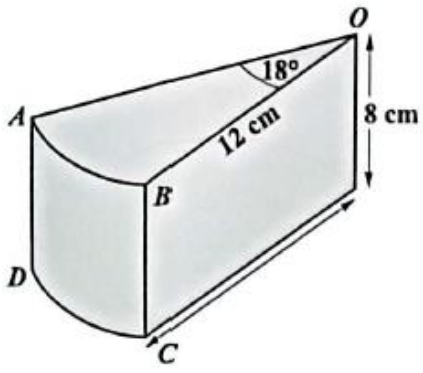
(a) A piece of wire, 61 cm long, is bent to form a sector, as shown in the diagram below. The sector of the circle, OPQ, has centre O and a radius of 18 cm.



(i) Show that the value of x is approximately 80°.

(ii) Calculate the area enclosed by the wire.

(b) A cylindrical block of cheese has a radius of 12 cm and a height of 8 cm. The cheese is divided into equal slices. The uniform cross-section of a slice of the cheese is a sector whose angle is 18°, as shown in the diagram below.



(i) Calculate the length of the arc AB.

(ii) Determine the area of the curved face, $ABCD$.

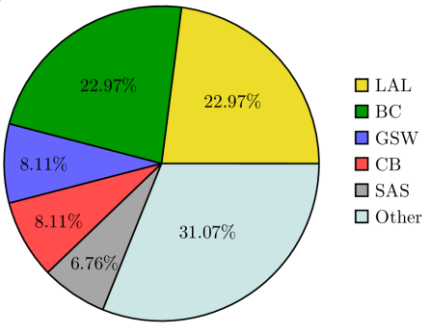
(iii) Given that the area of OAB is 22.6 cm^2, calculate the volume of the ENTIRE block of cheese.

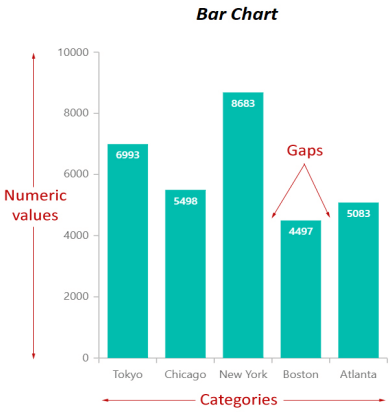
Section 5 – Statistics

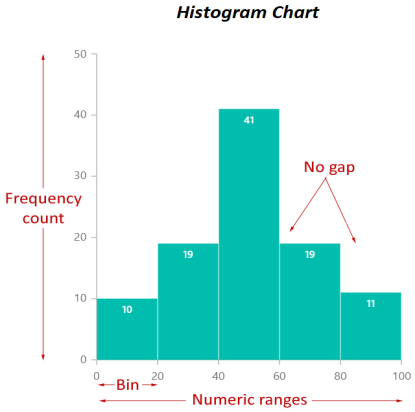
Basic definitions

- Population – The entire group being investigated
- Sample – A subset of the population
- Discrete data – Specific values only
- Continuous data – Range of values
- Raw data – Unordered info
- Mean – Average value = $\frac{\sum x}{n}$ *(ungrouped)* or $\frac{\sum fx}{\sum f}$ *(grouped data)*
- Median – Middle value from a set of ordered values
- Mode – Most frequent value
- Probability – Chances of an event occurring
- Standard deviation – Gives a spread of the data. (*how far away from mean)*

Types of statistical charts:







To plot histograms, we need the class boundaries as shown below:

Class Limits	Class boundaries	Tally	Frequency
318 – 335	317.5 – 335.5	||||	4
336 – 353	335.5 – 353.5	𝍸	5
354 – 371	353.5 – 371.5	||	2
372 – 389	371.5 – 389.5	|||	3
390 – 407	389.5 – 407.5	|||	3
408 – 425	407.5 – 425.5	||	2
426 – 443	425.5 – 443.5	|	1

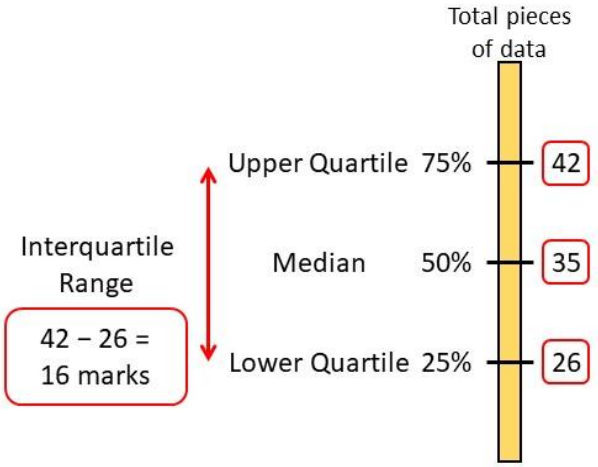
To construct a cumulative frequency graph and read off the Quartiles we do the following:

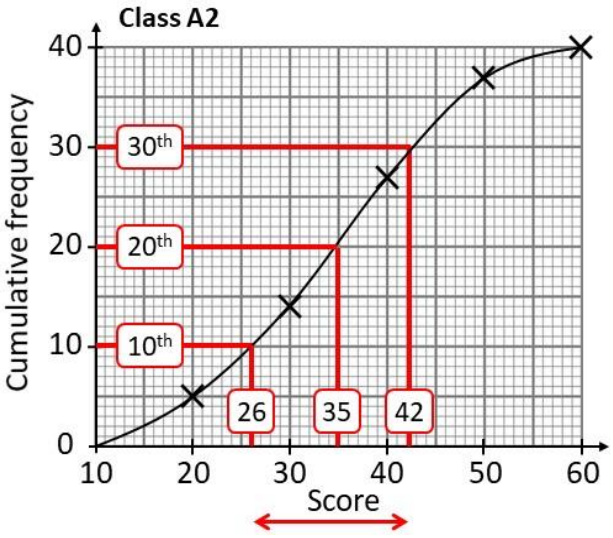
A cumulative frequency graph allows us to **measure the spread** of data.

By finding the value at each **quartile** (quarter) of the data, we can find the **median** and the spread of the **middle 50%**.

This measure eliminates extreme values.

Score	Students (Frequency)	Cumulative Frequency
11-20	5	5
21-30	9	14
31-40	13	27
41-50	10	37
51-60	3	40





Quartiles:

- Lower Quartile, Q_1 = $\frac{1}{4}(n+1)^{th}$ term
- Median, Q_2 = $\frac{1}{2}(n+1)^{th}$ term
- Upper Quartile, Q_3 = $\frac{3}{4}(n+1)^{th}$ term
- Inter Quartile Range = $Q_3 - Q_1$
- Semi-Inter Quartile Range = $\frac{Q_3 - Q_1}{2}$

Jan 2021 #5

5. (a) Sixty students took an algebra test, which comprised 15 multiple choice questions. The number of correct answers that each student obtained is recorded in the table below.

Number of Correct Answers	Number of Students
8	6
9	14
10	2
11	6
12	2
13	11
14	9
15	10

Using the table, determine

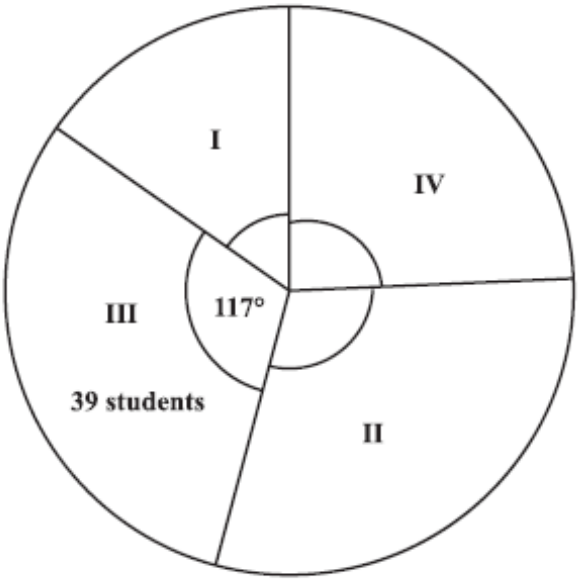
(i) the number of students who had exactly 13 correct answers

(ii) the modal number of correct answers

(iii) the median number of correct answers

(iv) the probability that a student chosen at random had at **least** 12 correct answers.

(b) A group of students wrote a Physics examination. Each of the students achieved a Grade I, II, III or IV. The pie chart below shows the results.



Thirty-nine students achieved a Grade III.

(i) Determine the TOTAL number of students who wrote the examination.

(ii) The ratio of the number of students who achieved a Grade I, II or IV is 2:4:3. A student passed the examination if he/she achieved a Grade I, II or III.

How many students passed the examination?

(iii) Determine the value of the angle for the sector representing Grade I in the pie chart.

May/June 2021 #5

5. (a) One hundred students were surveyed on the amount of money they spent on data for their cellphones during a week. The table below shows the results as well as the midpoint for each class interval.

Amount Spent ($)	Number of Students (f)	Midpoint ($) ($x$)
$50 < x \leq 60$	7	55
$60 < x \leq 70$	11	65
$70 < x \leq 80$	31	75
$80 < x \leq 90$	29	85
$90 < x \leq 100$	22	95

Using the table,

(i) a) determine the modal class of the amount of money spent

b) calculate an estimate of the mean amount of money spent, giving your answer correct to 2 decimal places.

(ii) Damion reports that the median amount spent is $84. Briefly explain why Damion's report could be correct.

(b) The two-way/contingency table below gives information on the mode of transportation to school for 100 students.

	Walk	Cycle	Drive	Total
Boy	15		14	48
Girl		18	26	
Total	23		40	100

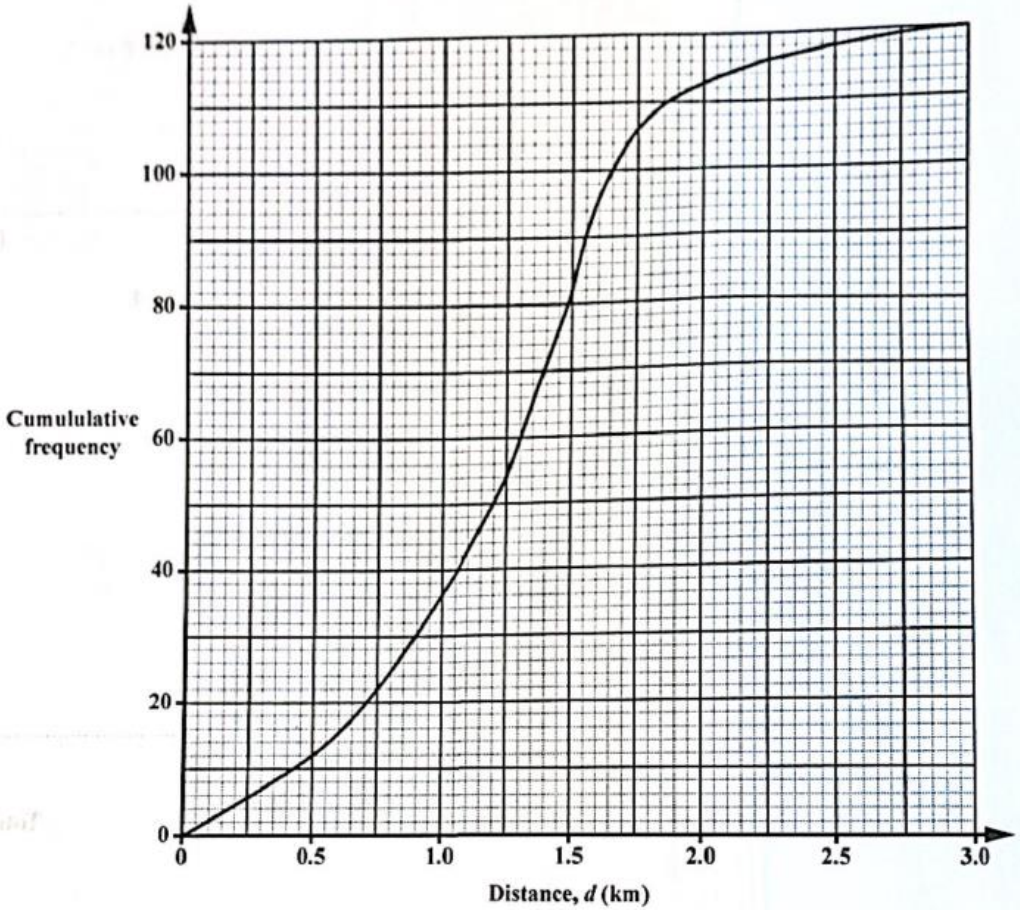
(i) Complete the table by inserting the missing values. **(2 marks)**

(ii) A student is selected at random. What is the probability that he/she was being driven to school on that day?

(iii) One of the girls is selected at random. What is the probability that she did NOT cycle to school?

Jan 2022 #5

5. The cumulative frequency diagram below shows information about the distance, d km, that each of 120 students walks to school on a particular day.



(a) How many students walked AT MOST 1 km to school on that day?

(b) Using the cumulative frequency diagram, determine an estimate of

(i) the median

(ii) the lower quartile

(iii) the interquartile range.

(c) A student is chosen at random. What is the probability that the student walked for **more than** 1.5 km to school that day?

(d) Complete the table below and use the information to calculate an estimate of the mean distance walked by the students on that day.

Distance, d (km)	Midpoint (x)	Number of Students (f)	$f \times x$
$0 < d \leq 0.5$	0.25	12	3.0
$0.5 < d \leq 1.0$	0.75	24	18
$1.0 < d \leq 1.5$	1.25	46	57.5
$1.5 < d \leq 2.0$	_____	_____	_____
$2.0 < d \leq 2.5$	2.25	_____	_____
$2.5 < d \leq 3.0$	2.75	2	5.5

May/June 2022 #5

5. A school nurse records the height, h cm, of each of the 150 students in Class A who was vaccinated. The table below shows the information.

Height, h (cm)	Number of Students (f)
$60 < h \leq 80$	4
$80 < h \leq 100$	20
$100 < h \leq 120$	35
$120 < h \leq 140$	67
$140 < h \leq 160$	20
$160 < h \leq 180$	4

(a) Complete the table below and use the information to calculate an estimate of the mean height of the students. Give your answer correct to 1 decimal place.

Height, h (cm)	Number of Students (f)	Midpoint (x)	$f \times x$
$60 < h \leq 80$	4	70	280
$80 < h \leq 100$	20	90	1 800
$100 < h \leq 120$	35	110	3 850
$120 < h \leq 140$	67	_____	_____
$140 < h \leq 160$	20	150	3 000
$160 < h \leq 180$	4	170	680

(b) In Class B, the mean height of the students is 123.5 cm, and the standard deviation 29.87. For Class A, the standard deviation is 21.38.

Using the information provided, and your response in (a), comment on the distribution of the heights of the students in both Class A and Class B.

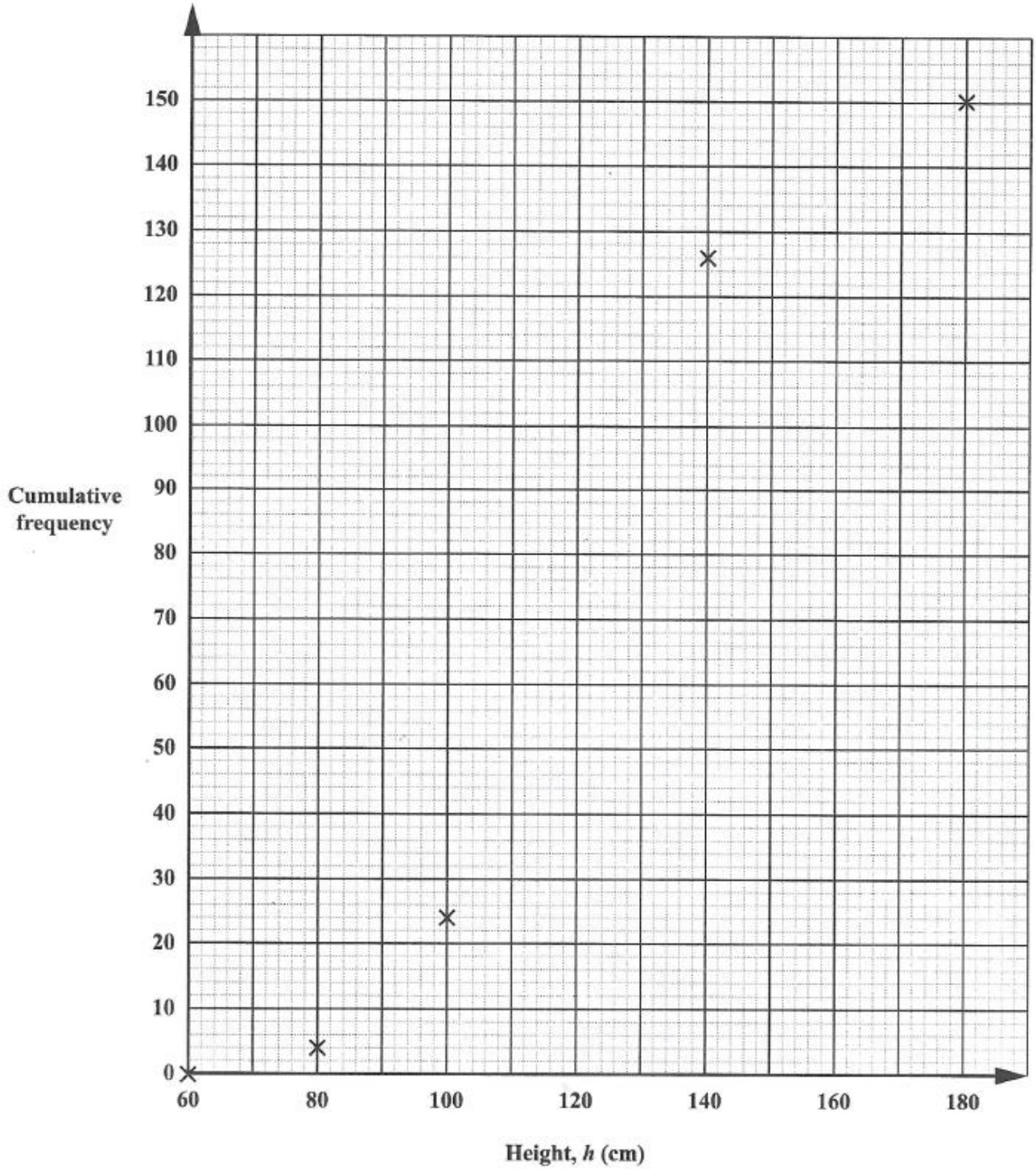
(c) (i) Complete the cumulative frequency table below and use the information to construct the cumulative frequency curve on the grid provided **on page 19**.

Height, h (cm)	Number of Students (f)	Cumulative Frequency
$60 < h \leq 80$	4	4
$80 < h \leq 100$	20	24
$100 < h \leq 120$	35	_____
$120 < h \leq 140$	67	126
$140 < h \leq 160$	20	_____
$160 < h \leq 180$	4	150

(ii) Use your cumulative frequency curve to find

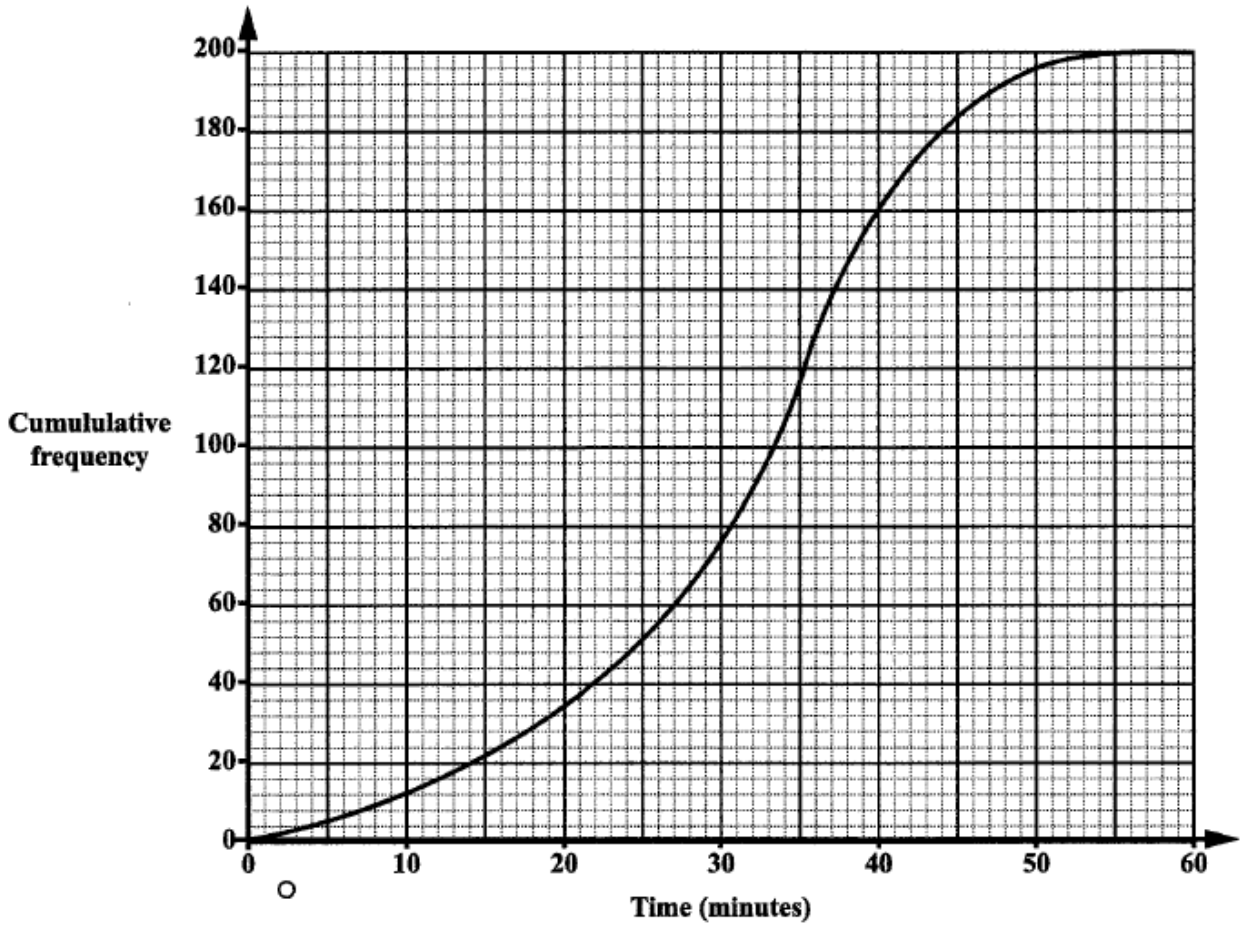
a) an estimate of the median height of the group of students

b) the probability that a student chosen at random would be taller than 130 cm.


Cumulative frequency
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
60
80
100
120
140
160
180
Height, h (cm)

Jan 2023 #5

5. The cumulative frequency curve below shows information about the times taken by 200 students to solve a Mathematics Olympiad problem.



(a) Using the cumulative frequency curve shown above, find an estimate for the

(i) number of students who took more than 50 minutes to solve the problem

(ii) median time taken to solve the problem

(iii) probability that a student chosen at random took **at most** 28 minutes to solve the problem.

(b) (i) Using the cumulative frequency curve **on page 13**, complete the table below.

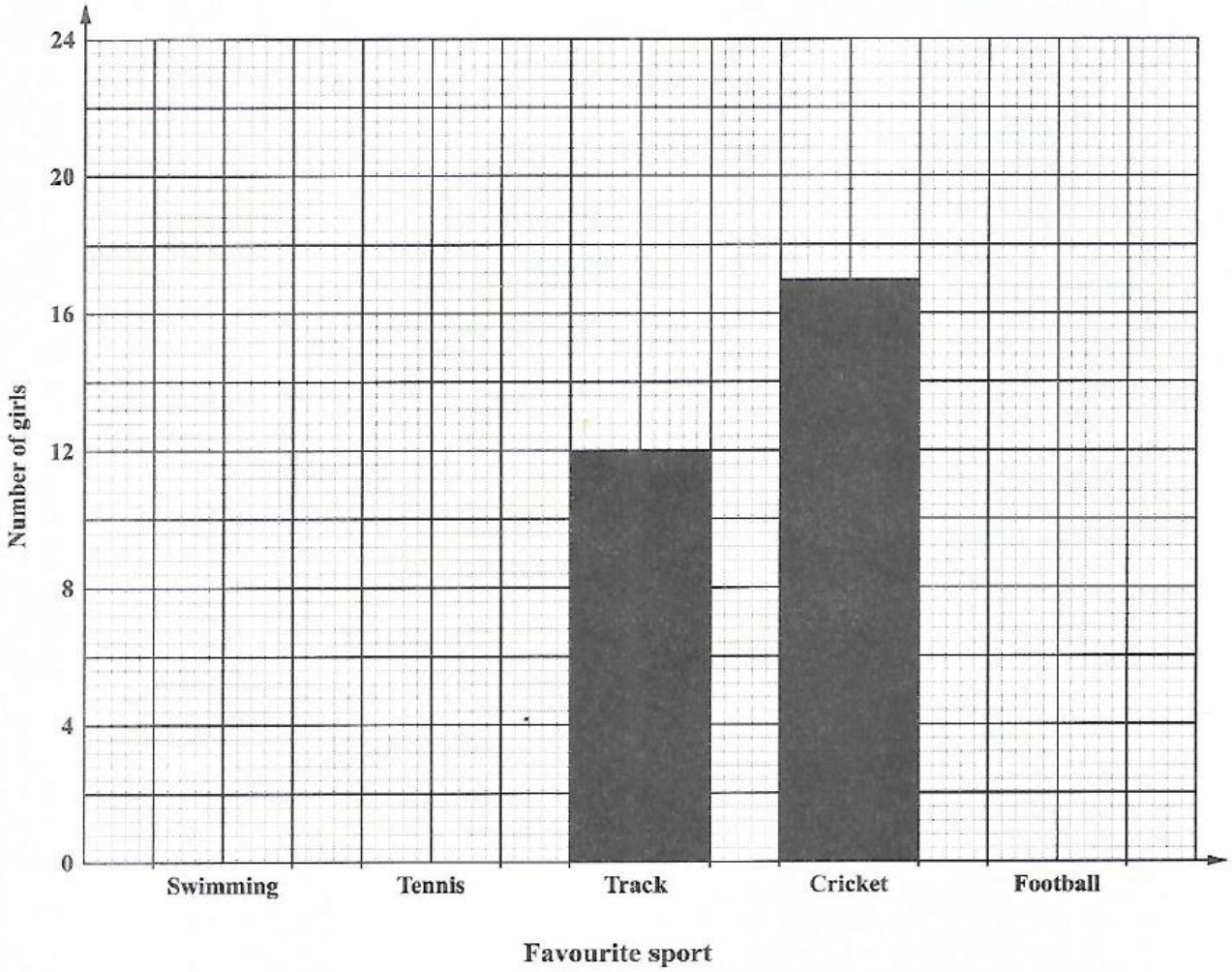
Time (minutes)	**Midpoint (x)**	**Frequency (f)**	**Frequency × Midpoint (fx)**
1 – 10	5.5	12	66
11 – 20	15.5	_____	_____
21 – 30	25.5	42	1071
31 – 40	35.5	84	2982
41 – 50	45.5	_____	_____
51 – 60	55.5	4	222

(3 marks)

(ii) Use the information in the completed table above to calculate an estimate of the average time taken by the students to solve the problem.

May/June 2023 #5

5. Each of 75 girls recorded the name of her favourite sport. The number of girls who chose track and cricket are shown on the bar chart below.



(a) How many **more** girls chose cricket than track as their favourite sport?

.....

(1 mark)

(b) Eleven girls recorded tennis as their favourite sport. For the remaining girls, the number who chose swimming compared to the number who chose football was in the ratio 2:3.

Use this information to complete the bar chart above. **(3 marks)**

(c) Determine the modal sport.

(d) One of the girls is selected at random. What is the probability that she chose NEITHER track NOR cricket as her favourite sport?

(e) The information on the favourite sport of the 75 girls is to be shown on a pie chart. Calculate the sector angle for football.

Jan 2024 #5

5. The mass, m, in kilograms, of 120 newborns at a hospital is recorded in the table below.

Mass (m kg)	**Frequency (f)**
$2.6 < m \leq 3.5$	7
$3.5 < m \leq 4.4$	18
$4.4 < m \leq 5.3$	30
$5.3 < m \leq 6.2$	29
$6.2 < m \leq 7.1$	28
$7.1 < m \leq 80$	8

(a) (i) State the modal class.

(ii) Complete the table below and calculate an estimate of the mean mass of the 120 newborns.

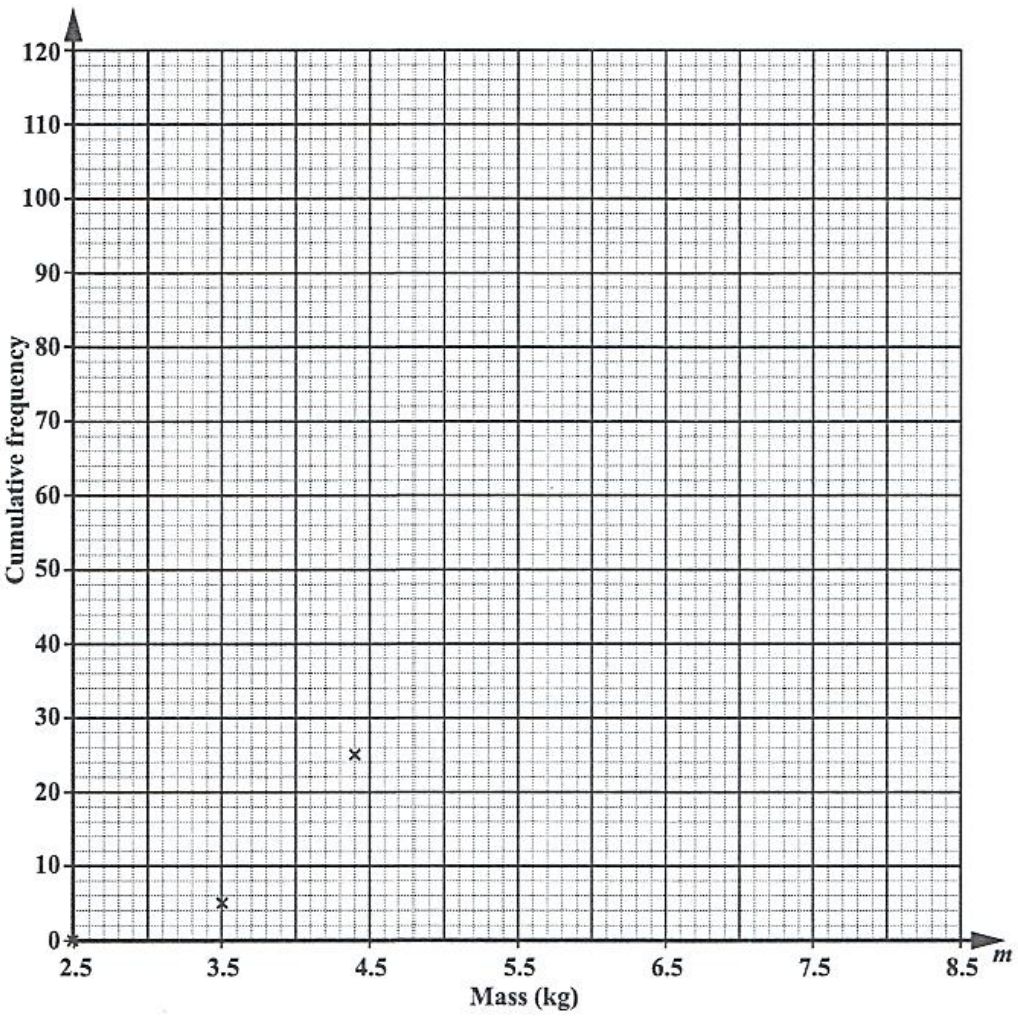
Mass (m kg)	**Midpoint (x)**	**Frequency (f)**	**Frequency × Midpoint (fx)**
$2.6 < m \leq 3.5$	3.05	7	21.35
$3.5 < m \leq 4.4$	3.95	18	71.1
$4.4 < m \leq 5.3$	4.85	30	145.5
$5.3 < m \leq 6.2$	_____	29	_____
$6.2 < m \leq 7.1$	6.65	28	186.2
$7.1 < m \leq 8.0$	_____	8	_____

(iii) One newborn is chosen at random from the hospital. Find the probability that the newborn has a mass greater than 5.3 kg.

(b) (i) Complete the cumulative frequency table shown below.

Mass (m kg)	Cumulative Frequency
$m \leq 3.5$	7
$m \leq 4.4$	25
$m \leq 5.3$	55
$m \leq 6.2$	_____
$m \leq 7.1$	_____
$m \leq 8.0$	_____

(ii) On the grid below, draw a cumulative frequency curve to show the information in the table in (b) (i) **on page 16**.



(iii) Use your diagram to find an estimate for the median mass of the newborns.

May/June 2024 #5

5. (a) Mr Morgan administered a spelling test to his class. The table below shows the number of words out of 10 that each student spelt correctly.

Number of Words	5	6	7	8	9	10
Frequency	8	4	2	2	3	4

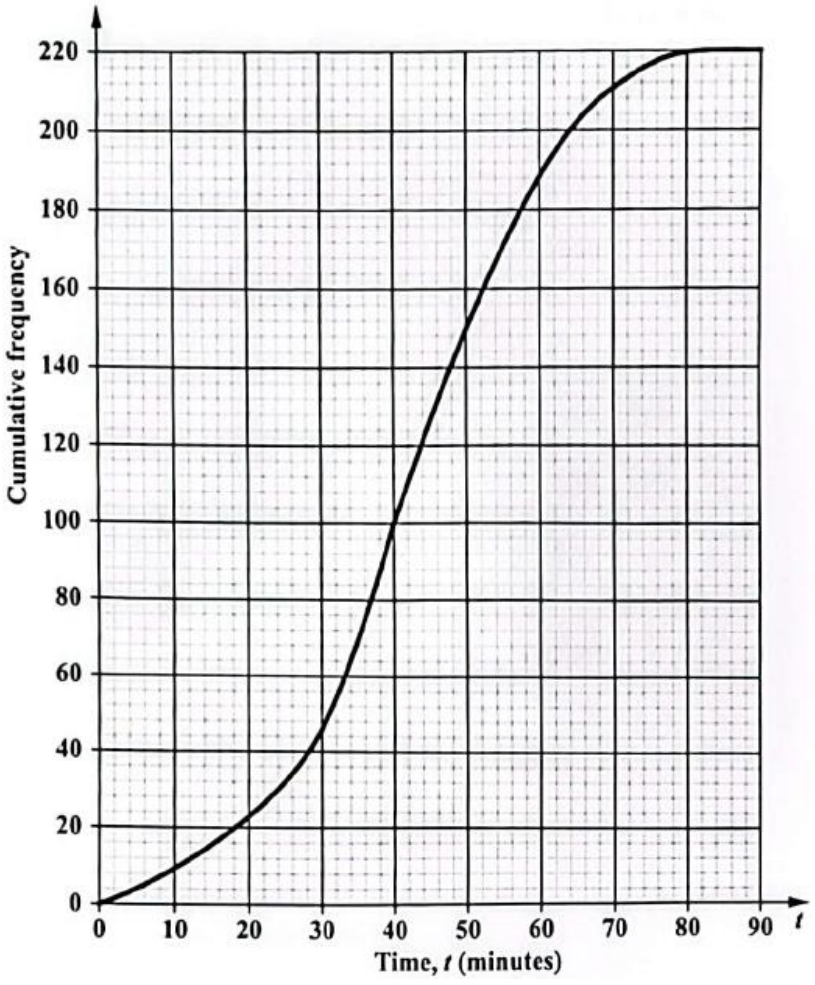
(i) For the data set shown above, state the

a) mode

b) median.

(ii) Calculate the mean number of words spelt correctly.

(b) The attendance officer at a particular school recorded the time, t, in minutes, taken by each student in a group to travel to school. The data collected is shown on the cumulative frequency curve below.

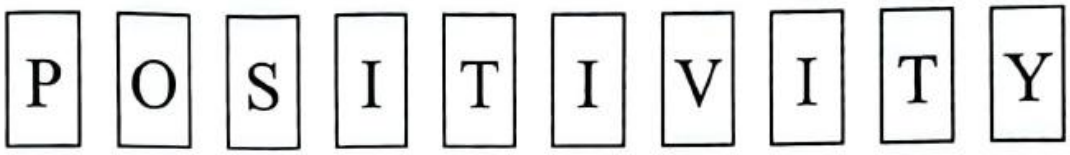


Using the cumulative frequency curve, find an estimate of

(i) the number of students who took at MOST 32 minutes to travel to school

(ii) the inter-quartile range.

(c) The letters in the word "POSITIVITY" are each written on separate cards and placed in a bag. Dacia picks 2 of these cards, at random, **with replacement**.



Find the probability that she picks the letter "I" then the letter "V".

Jan 2025 #5

5. (a) The table below shows the marks, out of 10, that 40 students in a class gained on an essay writing test.

Marks (x)	Number of Students (f)
4	3
6	9
7	8
8	7
9	8
10	5

(i) Calculate the students' mean score on the test.

(ii) Determine the

a) modal mark

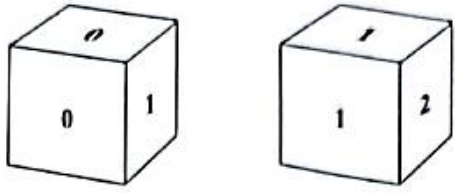
b) median mark.

(iii) Using the information in the table below, a pie chart is constructed to represent the marks students gained.

Marks (x)	Number of Students (f)
$3 \leq x \leq 4$	3
$5 \leq x \leq 6$	9
$7 \leq x \leq 8$	15
$9 \leq x \leq 10$	13

Calculate the angle for the sector representing the interval marks, $5 \leq x \leq 6$, in the pie chart.

(b) The diagram below shows two fair six-sided dice, P and Q.



P Q

The six numbers on Die P are 0, 0, 1, 1, 2, 3.
The six numbers on Die Q are 1, 1, 1, 2, 2, 3.
When a die is rolled, the score is the number on the top face.

(i) Die P is rolled once. What is the probability that the score is NOT 2?

(ii) Die Q is rolled **twice**. What is the probability that the score is 1 both times?

(iii) Die Q is rolled 72 times. Calculate an estimate of the number of times the score is 3.

(iv) Each die is rolled once. The product of the scores is recorded. The sample space diagram is shown below.

Die Q \ Die P	0	0	1	1	2	3
3	0	0	3	3	6	9
2	0	0	2	2	4	6
2	0	0	2	2	4	6
1	0	0	1	1	2	3
1	0	0	1	1	2	3
1	0	0	1	1	2	3

Find the probability that the product of the scores is 2 OR 3.

Section 6 – Algebra

Basic Algebra Rules:

- $x + x = 2x$
- $x - 2x = x\,(1 - 2) = -x$
- $x + y = x + y$

Indices Rules:

- $x^m \times x^n = x^{m+n}$
- $x^m \div x^n = x^{m-n}$
- $x^0 = 1$
- $(x^m)^n = x^{mn}$
- $x^{-m} = \frac{1}{x^m}$

Sign Rules:

- $- \times - = +$
- $- \times + = -$
- $+ \times - = -$
- $+ \times + = +$

N.B.
An expression has no equal sign (=), **but an equation has an equal sign.**

Simplifying:

When simplifying fractions:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm cb}{bd}$$

Expanding brackets:

$$(a + b)(a + b) = a^2 + 2ab + b^2$$

Factorization:

- $ab + ca = a(b + c)$
- $x^2y + y^2x = xy\,(x + y)$
- $px^2 + qx + apx + aq = (px^2 + qx) + (apx + aq)$
  $= x(px + q) + a(px + q)$
  $= (px + q)(x + a)$
- $a^2 - b^2 = (a - b)(a + b)$

Factorizing a Quadratic Expression: $ax^2 + bx + c$

1. Obtain a, b and c.

2. Get two numbers that when multiplied give ac and when added gives b.

3. Replace b with those two numbers then factorize.

Solving:

1. When given $\frac{a}{b} = \frac{c}{d}$, cross multiply to obtain $ad = bc$, then solve for the unknown.

2. When solving simultaneously, use elimination if there are 2 linear equations OR use the substitution method if there is a linear and a quadratic.

3. Direct Variation: $a \propto b$, therefore $a = kb$.

   Inverse Variation: $a \propto \frac{1}{b}$, therefore $a = \frac{k}{b}$.

Jan 2021 #2

2. (a) Factorize the following expression completely.

$$12n^2 - 4mn$$

(b) (i) Show that $\frac{x}{1-x} - 4x = \frac{x(4x-3)}{1-x}$.

(ii) Hence, solve the equation

$$\frac{x}{1-x} - 4x = 0.$$

(c) Make v the subject of the formula $p = \sqrt{5 + vt}$.

(d) The distance needed to stop a car, d, varies directly as the square of the speed, s, at which it is travelling. A car travelling at a speed of 70 km/h requires a distance of 40 m to make a stop. What distance is required to stop a car travelling at 80 km/h?

May/June 2021 #2

2. (a) Two quantities, n and T, are related as follows:

$$n = \sqrt{T}.$$

(i) Find the value of n when $T = 49$.

(ii) Make T the subject of the formula.

(b) Ally is x years. Jim is 5 years older than Ally and Chris is twice as old as Ally.

(i) Write expressions in terms of x for Jim's age and Chris' age.

Jim's age .....

Chris' age .....

(2 marks)

(ii) In **two years' time**, the product of Ally's age and Chris' age will be the same as the square of Jim's **present** age.

Show that the equation $x^2 - 4x - 21 = 0$ represents the information given above.

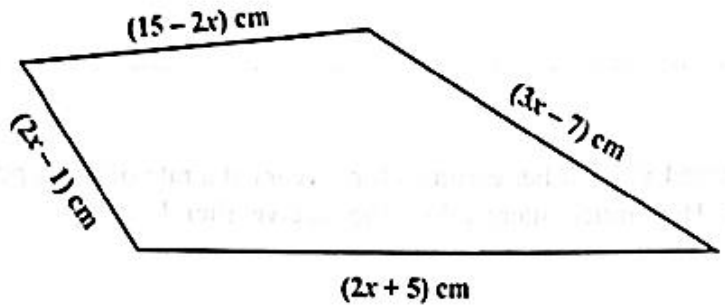
(iii) Calculate Ally's present age.

Jan 2022 #2

2. (a) Factorize completely

$3n^2 + 15np$.

(b) The diagram below shows a quadrilateral with the length of its sides written in terms of x.



(i) Write an expression, in terms of x, for the perimeter of the quadrilateral. Express your answer in its simplest form.

(ii) The perimeter of the quadrilateral is 32 cm.

Find the length of the longest side of the quadrilateral.

(c) Determine ALL the integer values of x which satisfy the inequality

$$-1 < \frac{2 - 4x}{3} < 5.$$

May/June 2022 #2

2. (a) (i) Factorize completely the following quadratic expression.

$5x^2 - 9x + 4$

(ii) Hence, solve the following equation.

$5x^2 - 9x + 4 = 0$

(b) Make v the subject of the formula.

$$w = \frac{5 + v}{v - 3}$$

(c) The height, h, of an object is directly proportional to the square root of its perimeter, p.

(i) Write an equation showing the relationship between h and p.

(ii) Given that $h = 5.4$ when $p = 1.44$, determine the value of h when $p = 2.89$.

Jan 2023 #2

2. (a) Simplify:

(i) $\left(x^3\right)^2$

(ii) $y^8 \div y^{-5}$

(b) (i) Factorize:

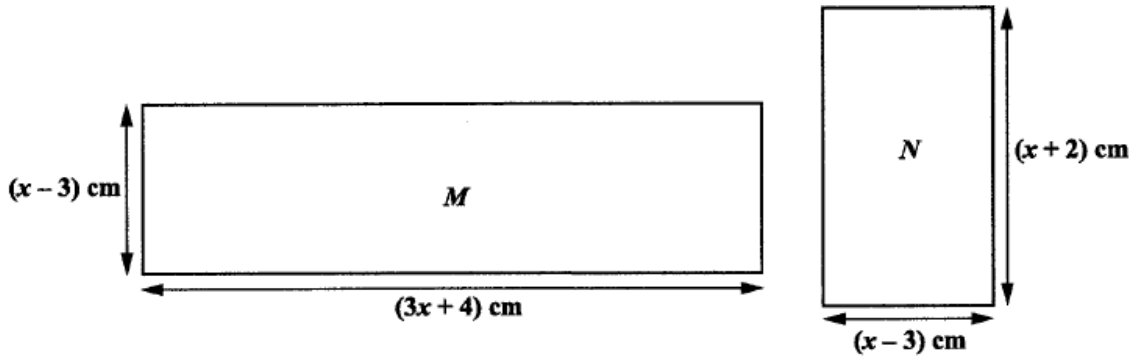
a) $xy - y^2$

b) $x^2 - y^2$

(ii) Hence, simplify the expression

$$\frac{xy - y^2}{x^2 - y^2}.$$

(c) The diagram below shows 2 rectangles, M and N, with their dimensions expressed in terms of x.



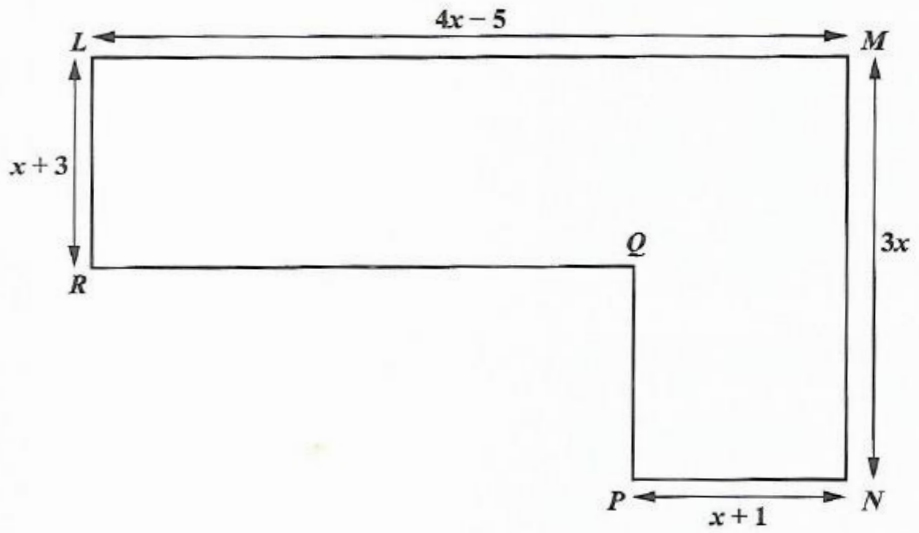
Given that the difference between the areas of the two rectangles is 64 cm^2, show that $x^2 - 2x - 35 = 0$.

May/June 2023 #2

2. (a) Simplify $\frac{4}{5x} \times \frac{15x}{16}$.

(b) Solve the inequality $12 - 4m \leq 5 - 8m$.

(c) The diagram below shows a compound shape, $LMNPQR$, made from two rectangles. The lengths in the diagram, which are written in terms of x, are in centimetres.



(i) Find an expression, in terms of x, for the length

a) PQ

b) RQ.

(ii) Given that the TOTAL area of the shape is 414 cm^2, show that $x^2 + x - 72 = 0$.

Jan 2024 #2

2. (a) Simplify

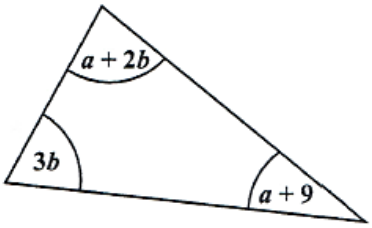
$$\frac{x^2 + 7x}{x^2 - 49}.$$

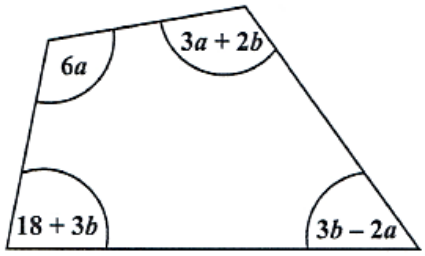
(b) Find the value of

(i) r, when $x^2 \times x^6 = x^r$

(ii) s, when $s^3 = 8$.

(c) The diagrams below show a triangle and a quadrilateral. All angles are in degrees and are written in terms of a and/or b.





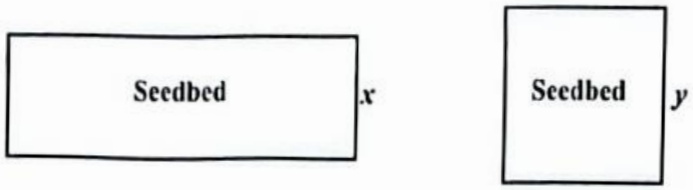
(i) For the triangle, show that $2a + 5b = 171$.

(ii) For the quadrilateral, show that $7a + 8b = 342$.

(iii) Solve the pair of simultaneous equations in (i) and (ii) to find the values of a and b. **Show all working.**

May/June 2024 #2

2. Laura needs to put mesh around **two** seedbeds to protect her seedlings. **Altogether**, she uses 60 m of mesh. One of the seedbeds is a rectangle and the other is a square, as shown in the diagram below.



The width of the rectangular seedbed is x metres. The length of the rectangular seedbed is 3 times its width. The length of a side of the square seedbed is y metres.

(a) Using the information given above, derive a simplified expression for y in terms of x.

(b) The area of the rectangular seedbed is **equal** to the area of the square seedbed.

(i) Use this information and your answer in (a) to write down a quadratic equation, in terms of x, and show that it simplifies to

$$x^2 - 60x + 225 = 0.$$

(ii) Solve the equation $x^2 - 60x + 225 = 0$ using the quadratic formula.

(iii) Calculate the TOTAL area of the **two** seedbeds.

Jan 2025 #2

2. (a) (i) Factorize EACH of the following algebraic expressions.

a) $x^2 - 49$

b) $x^2 + 2x - 35$

(ii) Hence, simplify the expression $\dfrac{x^2 - 49}{x^2 + 2x - 35}$.

(b) Rearrange the formula shown below to make m the subject.

$$s = k - m^2$$

(c) Lisa has \$56 to buy a total of **no more than** 70 red balloons and green balloons for her party.

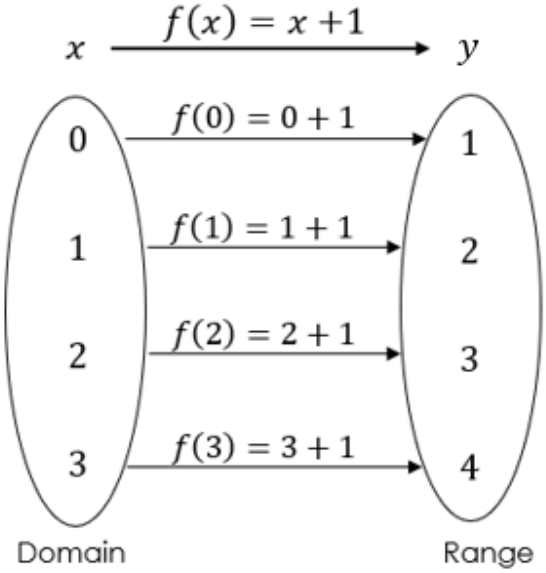
She buys more green balloons than red balloons but must buy at least 15 red balloons. Each red balloon costs \$0.75 and each green balloon costs \$0.50.

Let x and y represent the number of red balloons and the number of green balloons respectively. Write TWO inequalities in x and y, other than $x \geq 0$ and $y \geq 0$, to represent the information above.

(d) Given that y is inversely proportional to $(x - 2)$ and $x = 11$ when $y = 9$, find the value of y when $x = 29$.

Section 7 – Relations, Functions and Graphs

Functions:



The above diagram represents a 1-to-1 mapping.

Example:

If $f(x) = \frac{3x-1}{2}$

1. $f(1) = \frac{3(1)-1}{2} = \frac{2}{2} = 1$
2. $f(2) = \frac{3(2)-1}{2} = \frac{6-1}{2} = \frac{5}{2}$

Replace x with $3x$

3. $f(3x) = \frac{3(3x)-1}{2} = \frac{9x-1}{2}$

Replace x with $2x + 1$

4. $f(2x+1) = \frac{3(2x+1)-1}{2} = \frac{6x+3-1}{2} = \frac{6x+2}{2} = 3x + 1$

Composite Functions:

Composite functions represent functions of functions, this means we substitute one function into the next.

Example:

$f(x) = 2x - 1 \quad , \quad g(x) = \frac{x}{4}$

1. $f(g(x)) = f\left(\frac{x}{4}\right)$

$= 2\left(\frac{x}{4}\right) - 1$

$= \frac{x}{2} - 1$

2. $g(f(x)) = g(2x - 1)$

$= \frac{2x-1}{4}$

N.B.

$f^2(x) = f(f(x))$

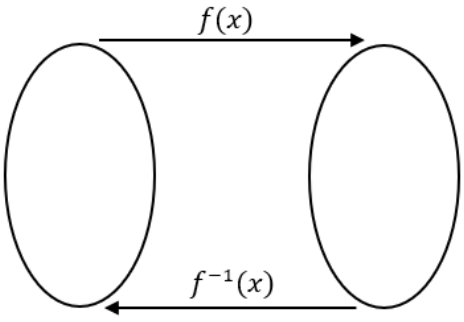
$= f(2x - 1)$

$= 2(2x - 1) - 1$

$= 4x - 2 - 1$

$= 4x - 3$

Inverse of a Function:



Steps to obtain the inverse of a function:

1. Let $y = f(x)$
2. Interchange x and y
3. Solve for y
4. $y = f^{-1}(x)$

Example: $f(x) = 2x - 1$

1. Let $y = 2x - 1$
2. Interchange x and y

   $x = 2y - 1$
3. Solve for y

   $2y - 1 = x$

   $2y = x + 1$

   $y = \frac{x+1}{2}$

   $\therefore f^{-1}(x) = \frac{x+1}{2}$

Co-ordinate Geometry:

- **Distance between two points =** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- **Mid-point =** $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

- **Gradient, m =** $\frac{y_2 - y_1}{x_2 - x_1}$

  N.B.

  - **Parallel lines** have equal gradients $(m_1 = m_2)$

  - The product of gradient for **perpendicular lines** is equal to -1.
    i.e. $(m_1 \times m_2 = -1)$

- **Equation of a line:** $y = mx + c$
  where m is the gradient and c is the y-intercept (cuts the y-axis).

- **To find the equation of a line:**
  1. Find the gradient of the line
  2. Obtain a point on the line
  3. Substitute in $c = y - mx$

N.B.
- Solving equations simultaneously gives the points of intersection of the equations.

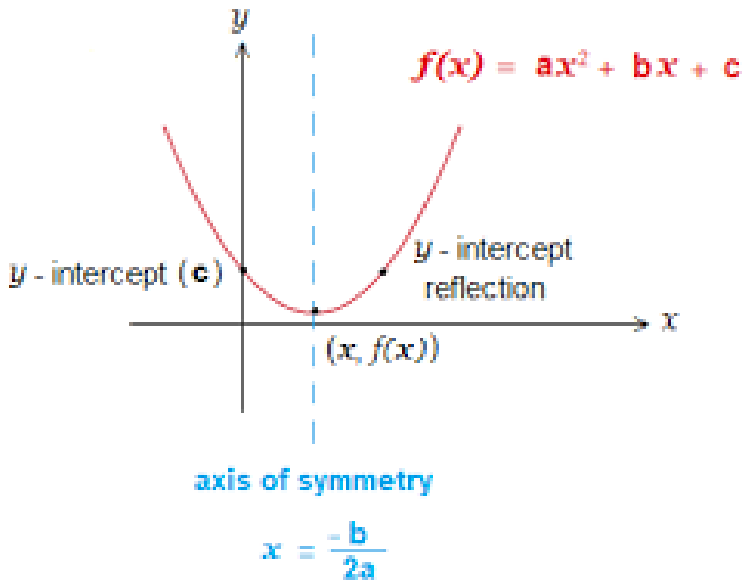
Quadratics:

- **General form:** $y = ax^2 + bx + c$ (The highest power of x is 2)

- **To complete the square:** $y = a(x + h)^2 + k$

  Where $h = \frac{b}{2a}$ and $k = c - ah^2$

- **To sketch a quadratic:**
    1. Shape: Minimum $(a > 0)$ Maximum $(a < 0)$
    2. Turning point: $(-h\ ,\ k)$
    3. Maximum or minimum value is always k.
    4. x – value which gives max. or min. value is $-h$.
    5. x – intercepts: Solve $ax^2 + bx + c = 0$
    6. y – intercept: $(0\ ,\ c)$



Inequalities:

- $<$ less than / fewer than
- $>$ greater than / more than
- $\leq$ at most / no more than
- $\geq$ at least / no less than

Solve inequalities like equations but change the inequality sign when divided by a negative.

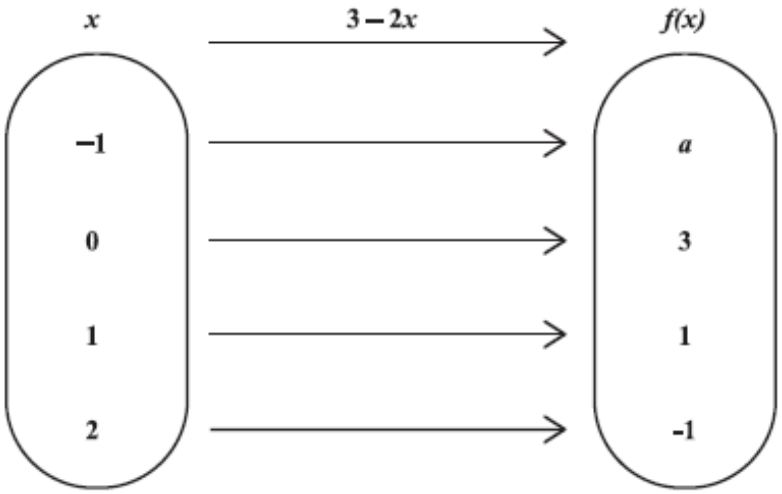
- For $<$ or $\leq$: shade below the line
- For $>$ or $\geq$: shade above the line

Jan 2021 #4

4. (a) The function f is defined as

$f: x \to 3 - 2x$.

(i) The diagram below shows the mapping diagram of the function, f. Determine the value of a.



(ii) Determine, in their simplest form, expressions for

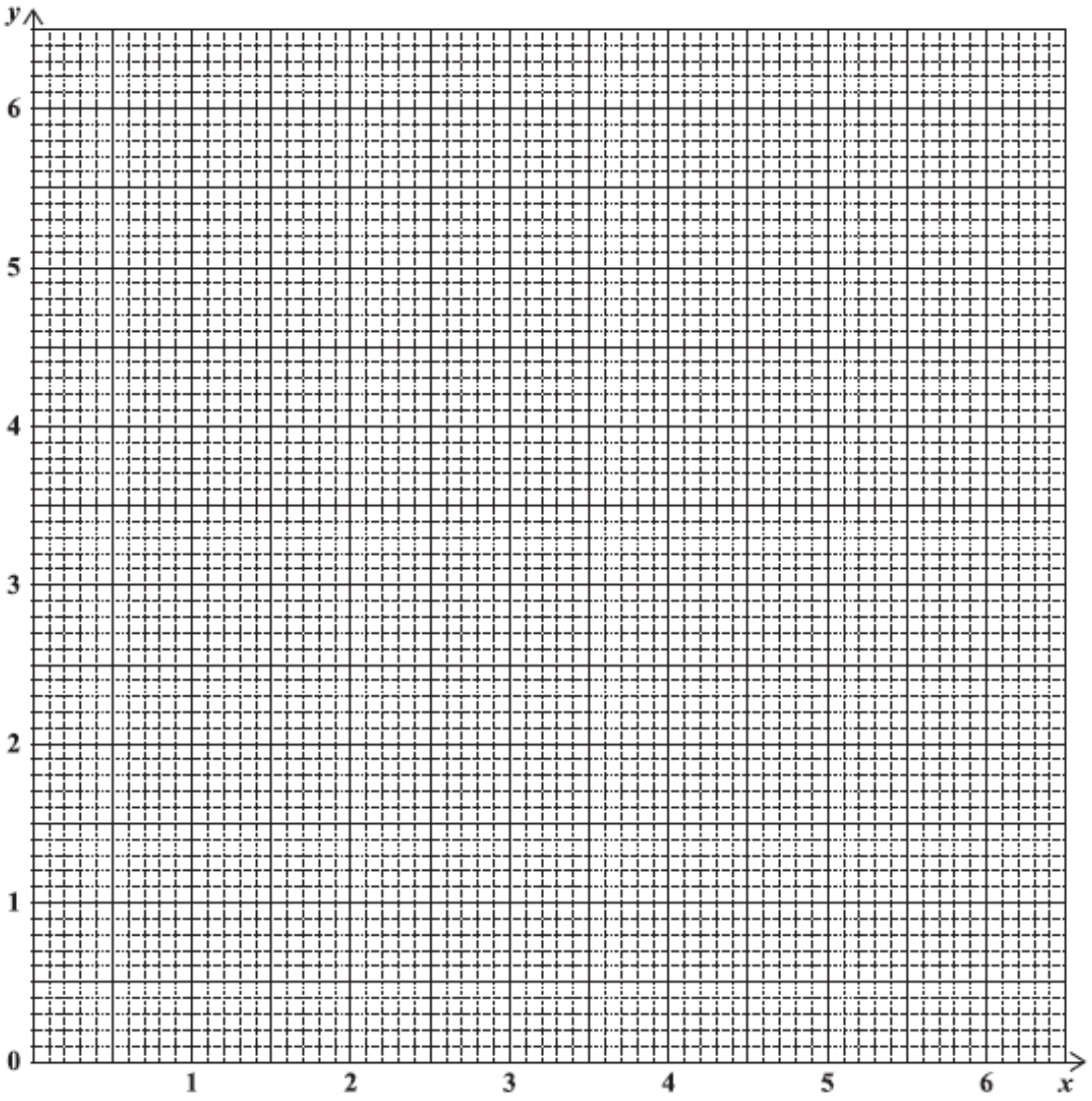
a) the inverse of the function f, $f^{-1}(x)$

b) the **composite** function $f^2(x)$.

(iii) State the value of $ff^{-1}(-2)$.

(b) (i) Using a ruler, draw the lines $x = \frac{1}{2}$, $y = x$ and $x + y = 5$, on the grid below.

(3 marks)



(ii) On the grid, label as R, the region where $x \geq \frac{1}{2}$, $y \geq x$ and $x + y \leq 5$. **(1 mark)**

Total 9 marks

Jan 2021 #8

8. (a) The straight line graph of $x = 5 - 3y$ intersects the x-axis at P and the y-axis at Q.

(i) Determine the coordinates of P and Q.

(ii) Calculate the length of PQ, giving your answer to 2 decimal places.

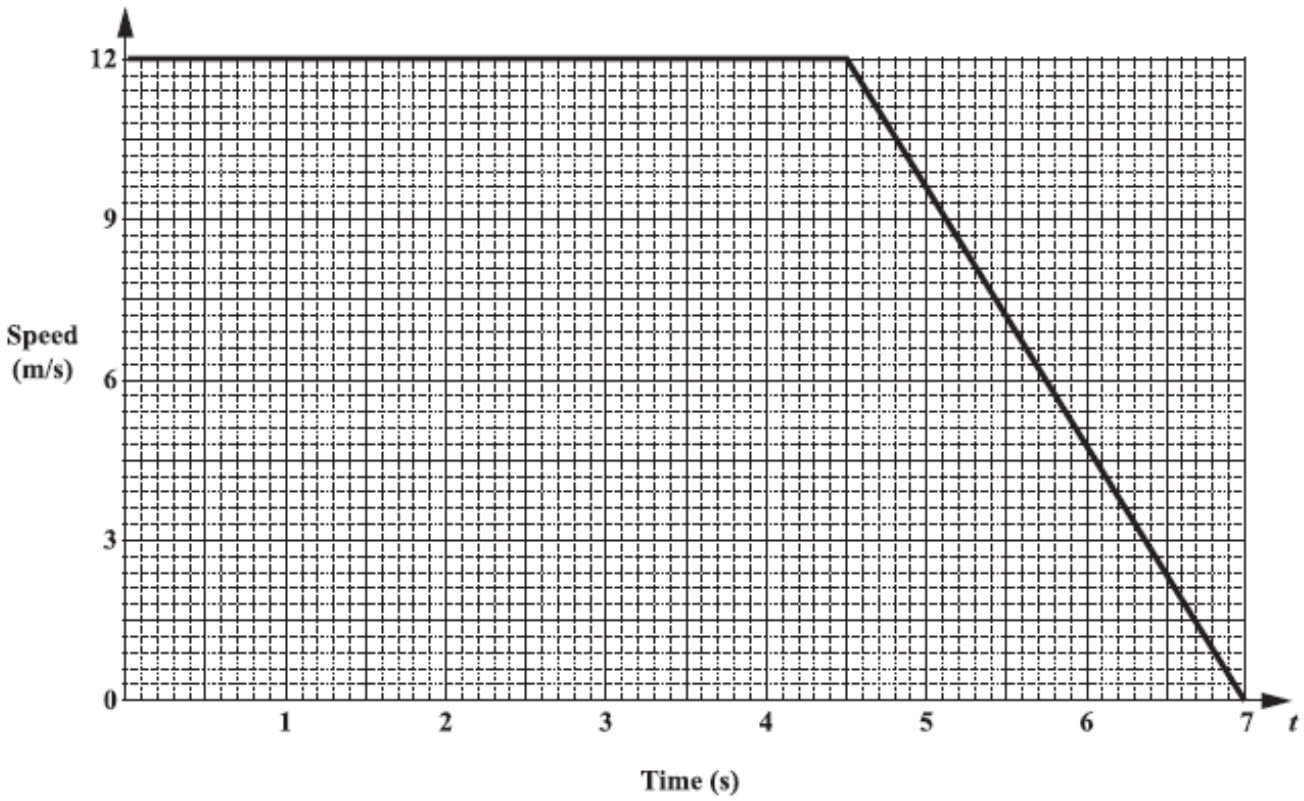
(iii) R is the midpoint of PQ. Determine the coordinates of R.

(b) The functions f and g are defined as follows

$$f : x \to 5 - x \text{ and } g : x \to x^2 - 2x - 1.$$

The graphs of $f(x)$ and $g(x)$ meet at points M and N. Determine the coordinates of the points M and N.

(c) Monty is cycling at 12 metres per second (m/s). After 4.5 seconds he starts to decelerate and after a further 2.5 seconds he stops. The speed–time graph is shown below.



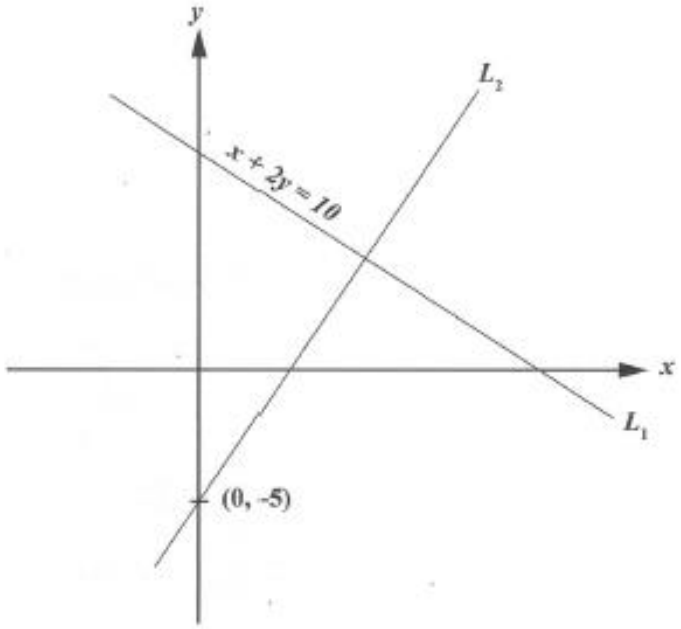
Calculate

(i) the constant deceleration

(ii) Monty's average speed over the 7 seconds.

May/June 2021 #4

4. (a) The diagram below shows two lines, L_1 and L_2. The equation of the line L_1 is $x + 2y = 10$. The line L_2 passes through the point $(0, -5)$ and is **perpendicular** to L_1.



(i) Express the equation of the line L_1 in the form $y = mx + c$.

(ii) State the gradient of the line L_1.

(iii) Hence, determine the equation of the line L_2.

(b) Given that $f(x) = \frac{1}{3}x + 4$ and $g(x) = \frac{3x}{x+1}$,

(i) determine the value of $f(9)$

(ii) calculate the value of $fg(-3)$

(iii) determine the value of x, for which $g(x) = \frac{5}{2}$.

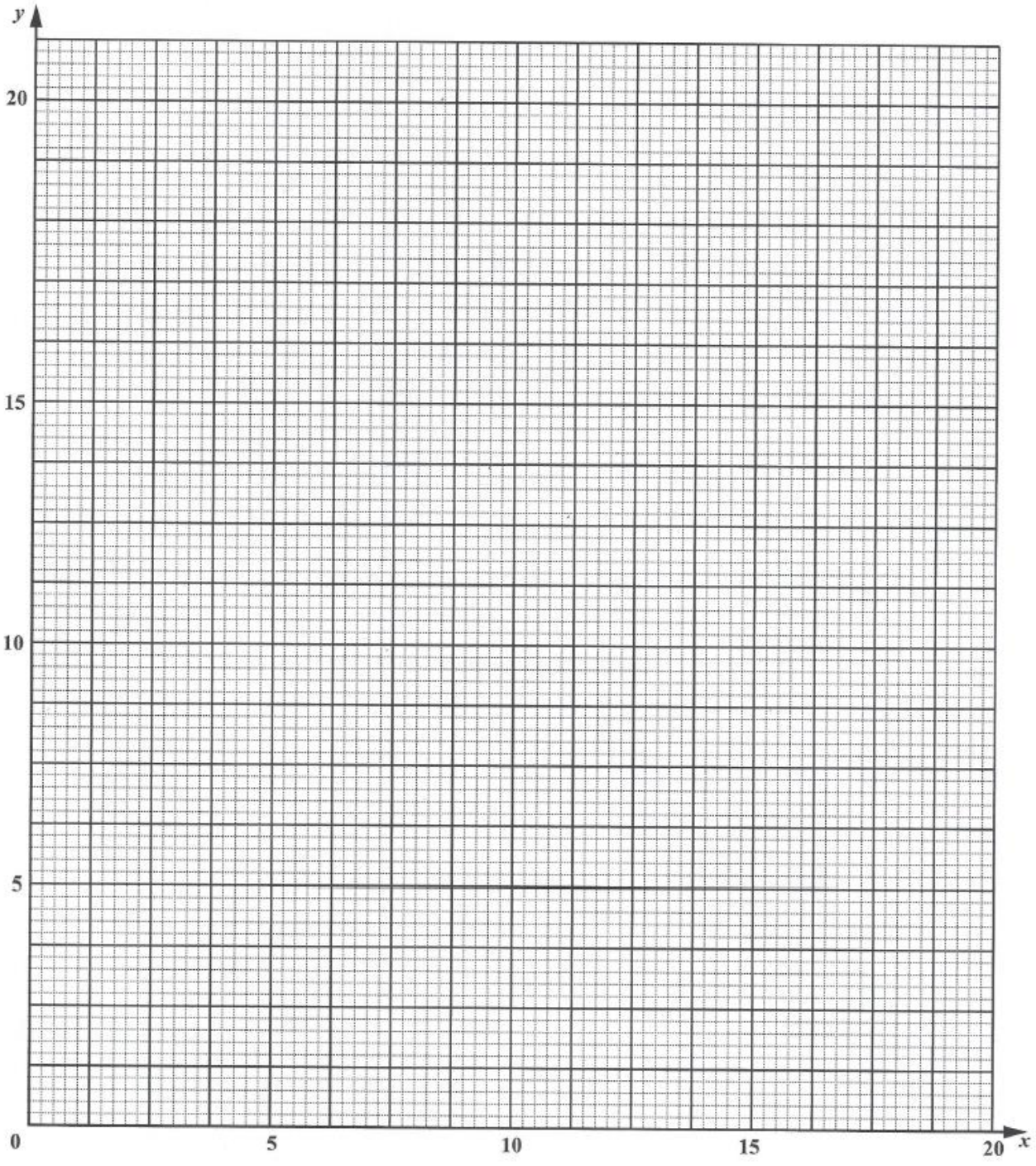
May/June 2021 #8

8. Marla buys 2 types of mobile phones, B-Flo and C-Flex, from a company to retail. One B-Flo mobile phone costs \$60 while one C-Flex costs \$80. She buys x number of B-Flo phones and y number of C-Flex phones.

(a) (i) Marla must **not** spend more than \$1 200. Write an inequality to represent this information.

(ii) The number of B-Flo phones must be greater than or equal to the number of C-Flex phones. Write down an inequality in x and y to show this information.

(iii) Represent the two inequalities **on page 22** on the grid shown below. Label as R the region which satisfies both inequalities.



(iv) The total number of mobile phones is represented by $x + y$. According to the graph **on page 23**, what is the largest possible value of $x + y$?

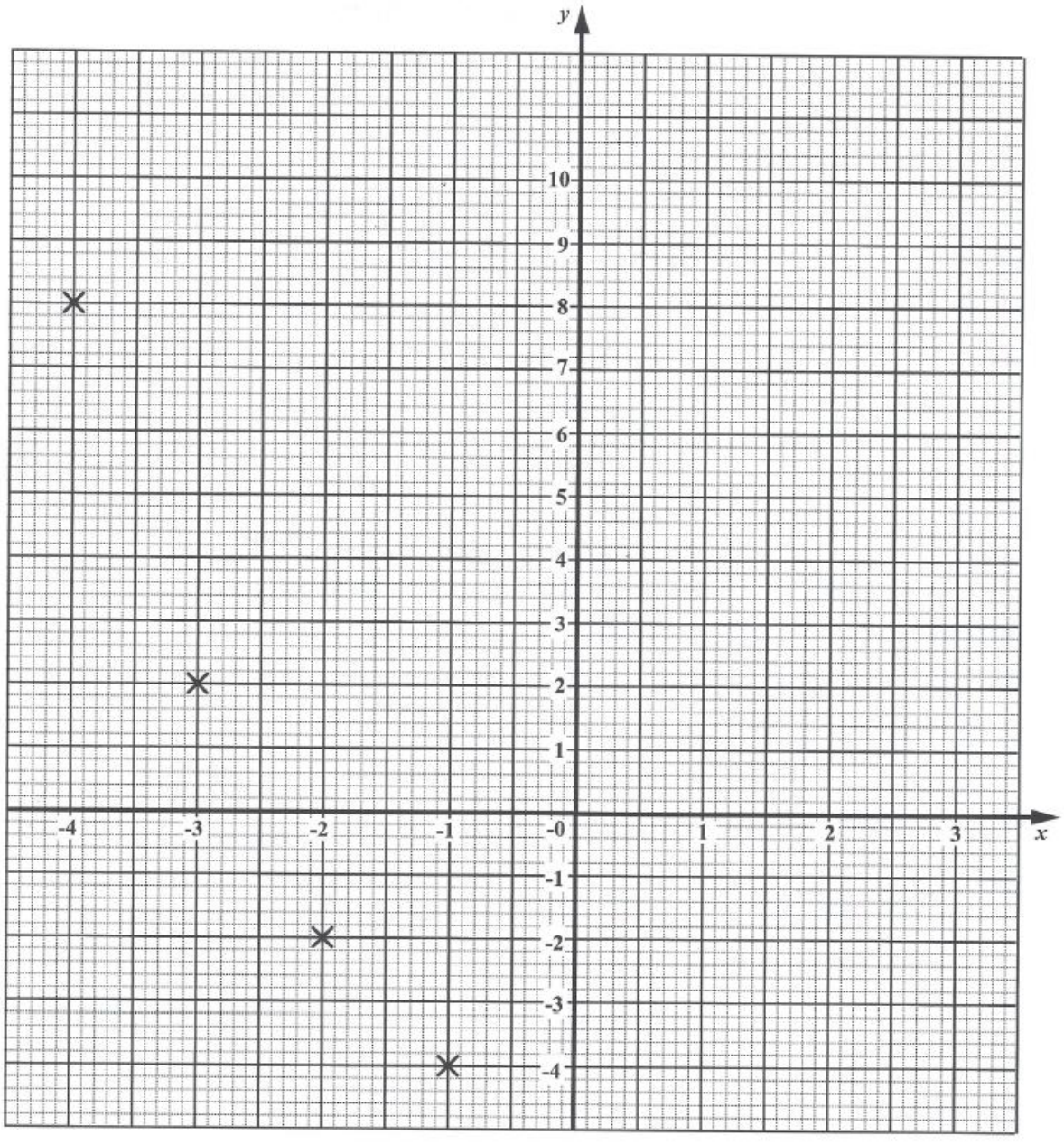
(b) The table below shows pairs of values for the function $y = x^2 + x - 4$.

x	–4	–3	–2	–1	0	1	2	3
y	8	2	–2	–4	–4	–2	2	8

(i) On the grid provided **on page 25**, plot the remaining 4 points and draw the graph of the function $y = x^2 + x - 4$ for $-4 \leq x \leq 3$. **(3 marks)**

(ii) Write down the maximum or minimum value of the function.

(iii) Using a ruler, draw the axis of symmetry on the graph **on page 25**. **(1 mark)**



Jan 2022 #4

4. Three functions f, g and h are defined as

$f(x) = 2x - 1$; $g(x) = 3x + 2$ and $h(x) = 5^x$.

(a) Find the value of

(i) $f\left(\frac{1}{2}\right)$

(ii) $h(0)$

(iii) $g^2(-3)$.

(b) Find $gf(x)$, giving your answer in its simplest form.

(c) (i) Find $g^{-1}(x)$.

(ii) Hence, or otherwise, determine the value of x when $g^{-1}(x) = 4$.

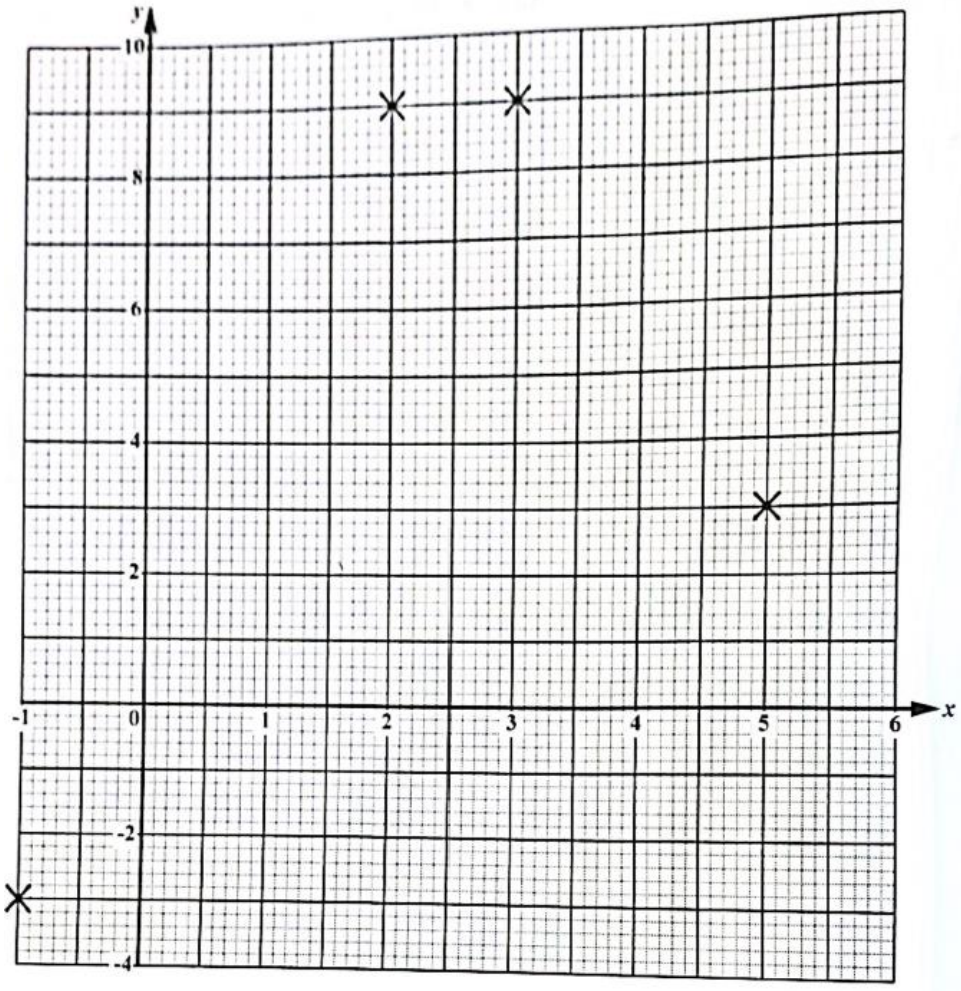
Jan 2022 #8

8. The function $f: x \rightarrow 3 + 5x - x^2$.

(a) (i) Complete the table of values for $f(x) = 3 + 5x - x^2$.

x	−1	0	1	2	3	4	5	6
$f(x)$	−3			9	9		3	

(ii) On the grid below, complete the graph of $f(x) = 3 + 5x - x^2$ for $-1 \leq x \leq 6$.



(b) (i) Write down the equation of the axis of symmetry of the graph of $f(x) = 3 + 5x - x^2$.

(ii) State the maximum value of the function.

(c) Write down the co-ordinates of the point where the line $y = 3 - \frac{1}{2}x$

(i) crosses the x-axis

(ii) crosses the y-axis.

(d) On the grid **on page 22**, draw the line $y = 3 - \frac{1}{2}x$.

(e) Using your graph, determine the solution to the equations

$y = 3 + 5x - x^2$

$y = 3 - \frac{1}{2}x$.

May/June 2022 #4

4. (a) The functions f and g are defined as follows:

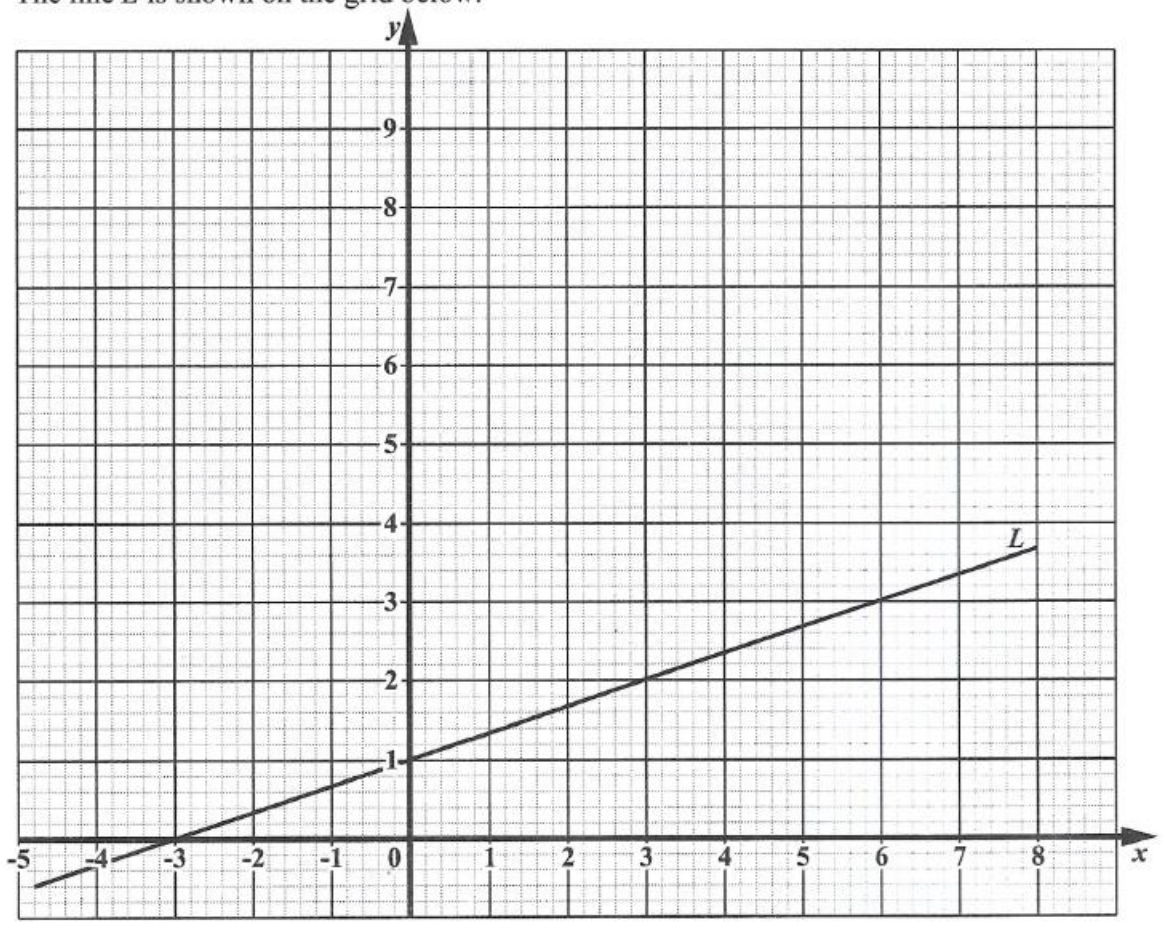
$$f(x) = 5x + 7 \text{ and } g(x) = 3x - 1.$$

For the functions given above, determine

(i) $g\left[\frac{1}{3}\right]$

(ii) $f^{-1}(-3)$.

(b) The line L is shown on the grid below.



(i) Write the equation of the line L in the form $y = mx + c$.

(ii) The equation of a different line, Q, is $y = -2x + 8$.

a) Write down the coordinates of the point where Q crosses the x-axis.

b) Write down the coordinates of the point where Q crosses the y-axis.

c) On the grid **on page 14**, draw the graph of the line Q. **(1 mark)**

(iii) Complete the statement below.

According to the graph, the solution of the system of equations consisting of L and Q is

May/June 2022 #8

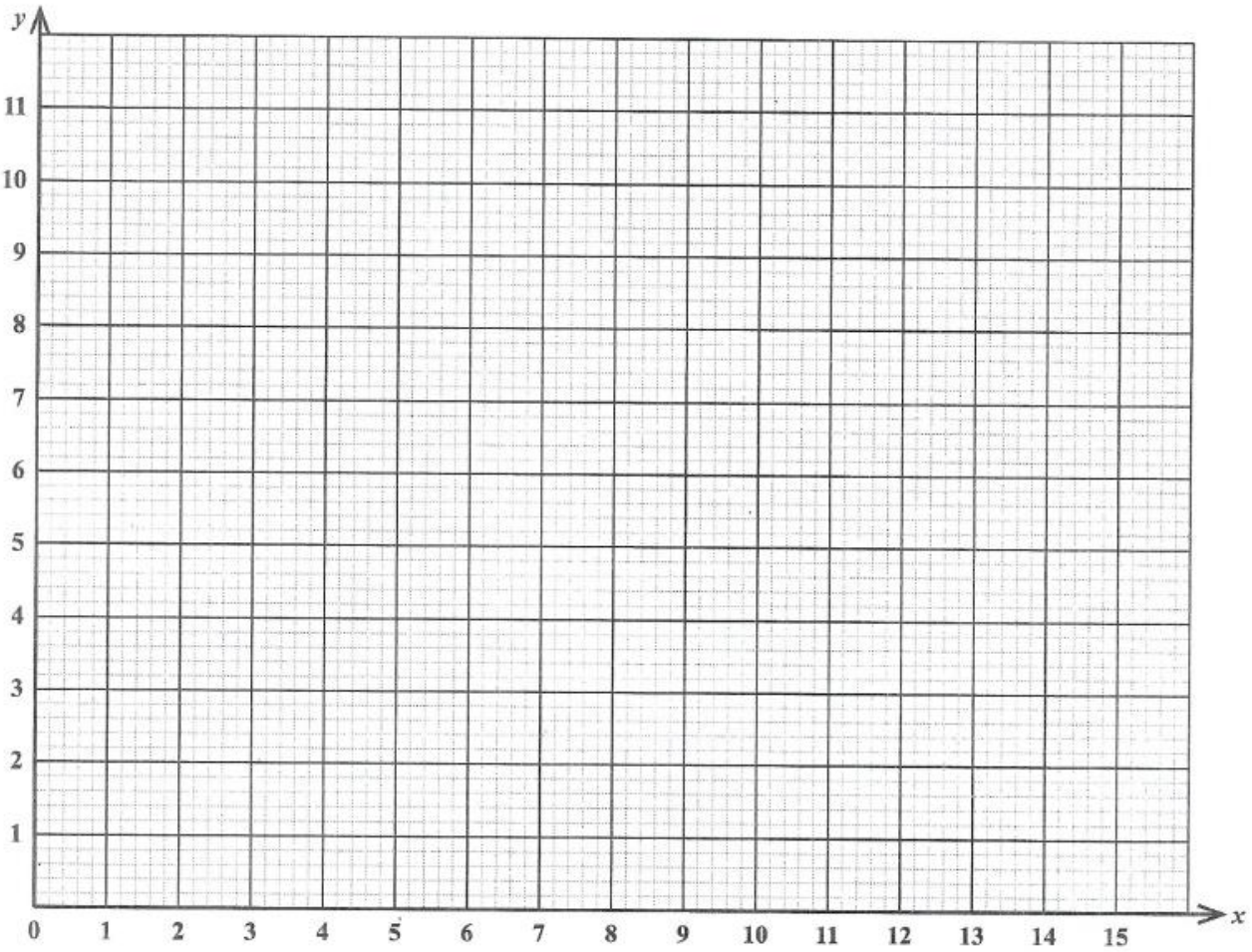
8. A rental company has x cars and y minivans. The company has at least 8 vehicles altogether. The number of minivans is less than or equal to the number of cars. The number of cars is no more than 9.

(a) Write down THREE inequalities, in terms of x and/or y, other than $x \geq 0$ and $y \geq 0$, to represent this information.

(b) A car can carry 4 people and a minivan can carry 6 people. There are at most 60 persons to be taken on a tour.

Show that $2x + 3y \leq 30$.

(c) On the grid below, plot the four lines associated with the inequalities in (a) and (b). Shade and label the region that satisfies ALL four inequalities R.

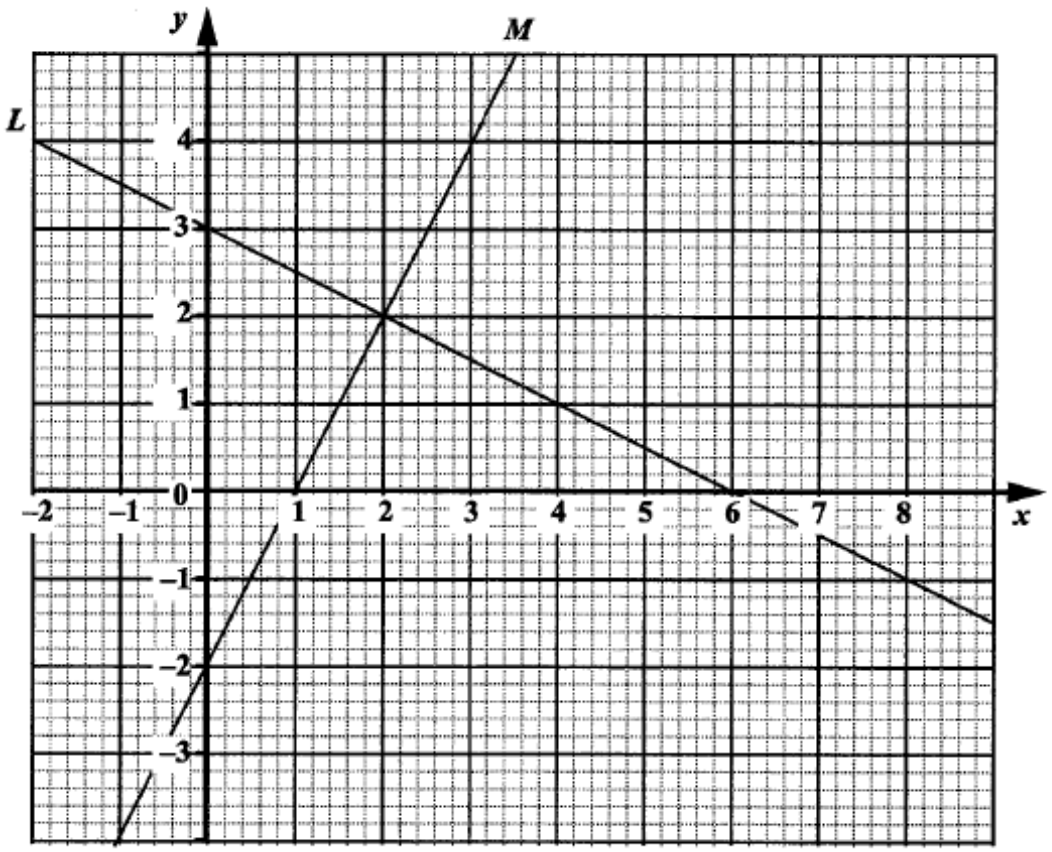


(d) (i) Determine the **two combinations** for the MINIMUM number of cars and minivans that can be used to carry EXACTLY 60 people on the tour.

(ii) The company charges \$75 to rent a car and \$90 to rent a minivan. Show that the MINIMUM rental cost for this tour is \$990.

Jan 2023 #4

4. Lines L and M are drawn on the square grid below.



(a) Write down the coordinates of the

(i) x-intercept of Line L

(ii) y-intercept of Line M.

(b) The equation of Line L is $x + 2y - 6 = 0$. Find the value of k given that the point $(9, k)$ lies on Line L.

(c) Find the equation of Line M, in the form $y = mx + c$.

(d) Show by calculation, that Line L and Line M are perpendicular.

(e) Line L and Line M represent the graph of a pair of simultaneous equations. Using the graph **on page 11**, write down the solution to the pair of simultaneous equations.

Jan 2023 #8

8. The functions f, g and h are defined as follows:

$$f(x) = 4x - 1,\ g(x) = x^2 - 5 \text{ and } h(x) = 3^x.$$

(a) Find

(i) $g(x-2)$, in its simplest form

(ii) $f^{-1}(11)$

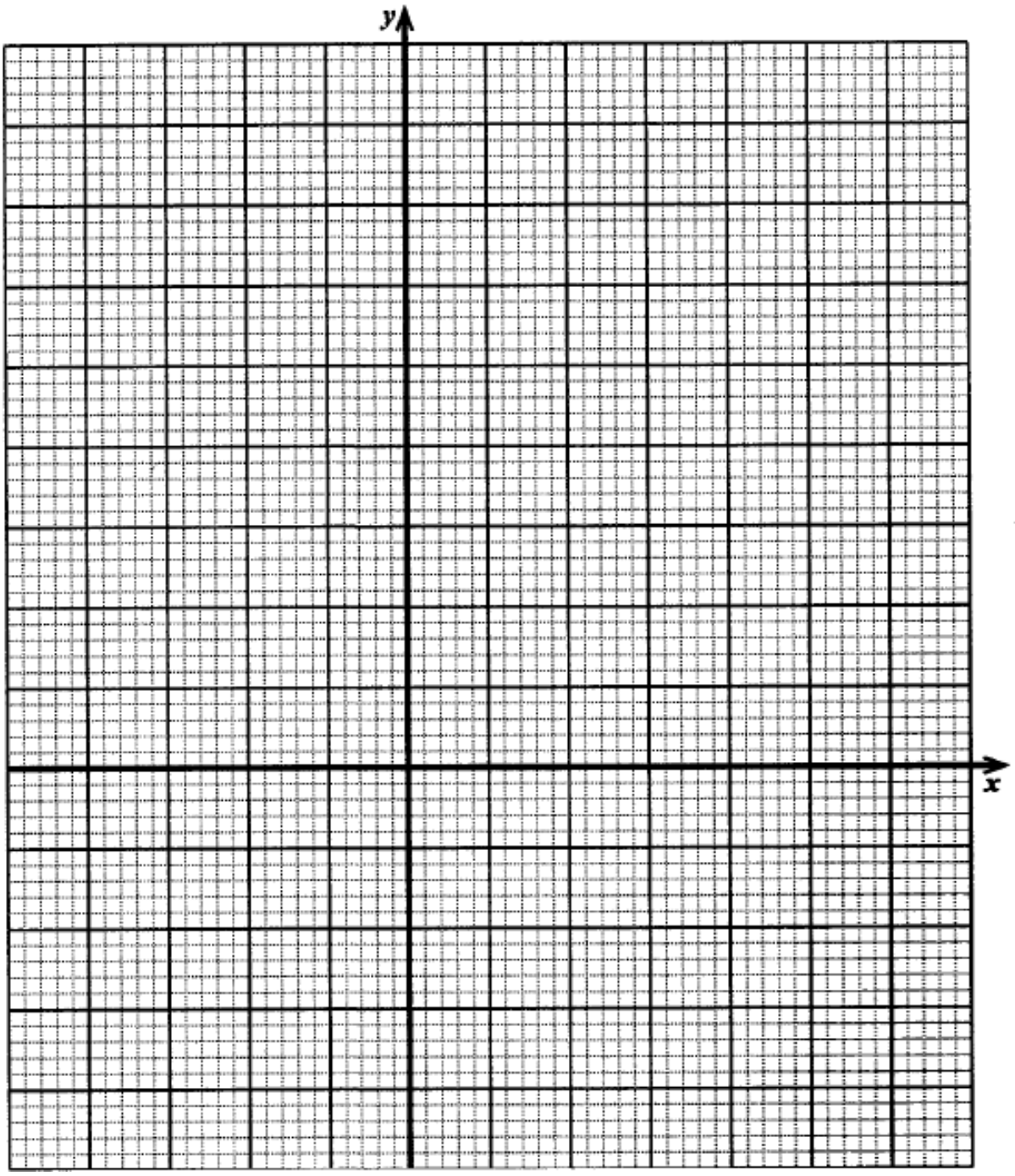
(b) Determine the value of $hh(1)$.

(c) The function f is defined as follows:

$$f : x \to x^2 - x - 2.$$

Complete the table below and plot the graph for the function $f(x) = x^2 - x - 2$ on the grid that follows.
(Use a scale of 2 cm to represent 1 unit on both axes.)

x	−2	−1	0	1	2	3
$f(x)$	_____	0	_____	−2	0	4


y
x

May/June 2023 #4

4. Consider the following functions.

$$f(x) = \frac{3}{x+2}, g(x) = 4x - 5 \text{ and } h(x) = x^2 + 1.$$

(a) (i) For what value of x is $f(x)$ undefined?

(ii) Find the value of

a) $g\left(\frac{1}{4}\right)$

b) $h(-3)$

c) $ff(0)$.

(b) Write an expression, in its **simplest** form, for $gh(x)$.

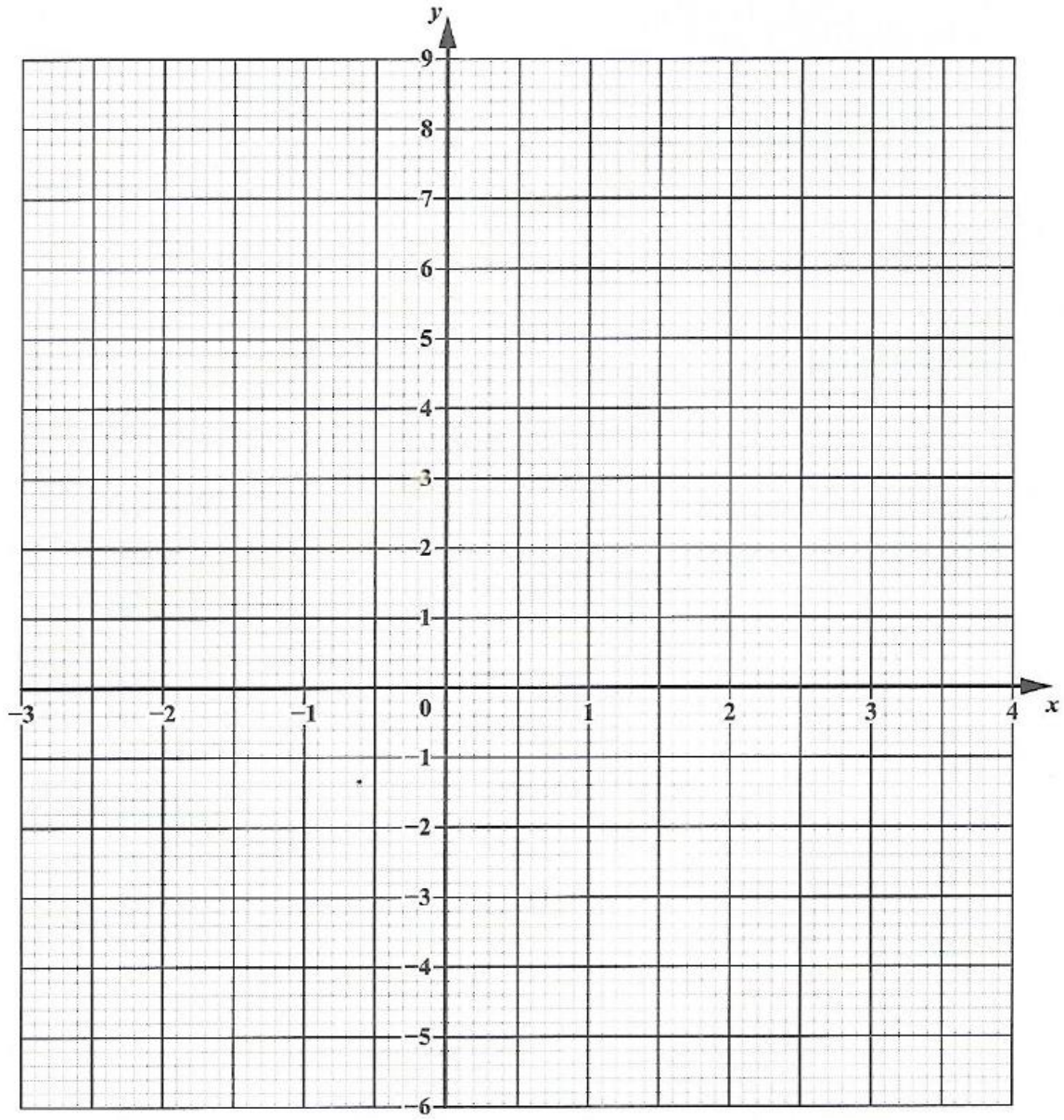
(c) Find $g^{-1}(-2)$.

May/June 2023 #8

8. (a) Complete the table for the function $y = -x^2 + x + 7$.

x	–3	–2	–1	0	1	2	3	4
y		1		7		5		–5

(b) On the grid below, draw the graph of $y = -x^2 + x + 7$ for $-3 \leq x \leq 4$.



(c) Write down the coordinates of the maximum/minimum point of the graph.

(d) Write down the equation of the axis of symmetry of the graph.

.....

(1 mark)

(e) Use your graph to find the solutions of the equation $-x^2 + x + 7 = 0$.

$x =$ ..... or $x =$ ..... **(2 marks)**

(f) (i) On the grid **on page 24**, draw a line through the points (–3, –1) and (0, 8).

(1 mark)

(ii) Determine the equation of this line in the form $y = mx + c$.

Jan 2024 #4

4. Consider the following functions.

$$f(x) = 2 - 5x \text{ and } h(x) = 5^x.$$

(a) Calculate the value of

(i) $f(4)$

(ii) $h(0)$

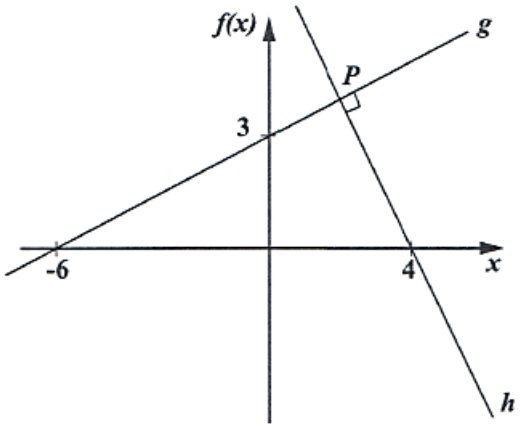
(iii) $fh(-2)$.

(b) Find $f^{-1}(x)$.

(c) Given that $ff(x) = a + bx$, determine the values of a and b.

Jan 2024 #8

8. (a) The diagram below shows the graphs of two functions on the same pair of axes. The lines g and h are perpendicular.



Determine the

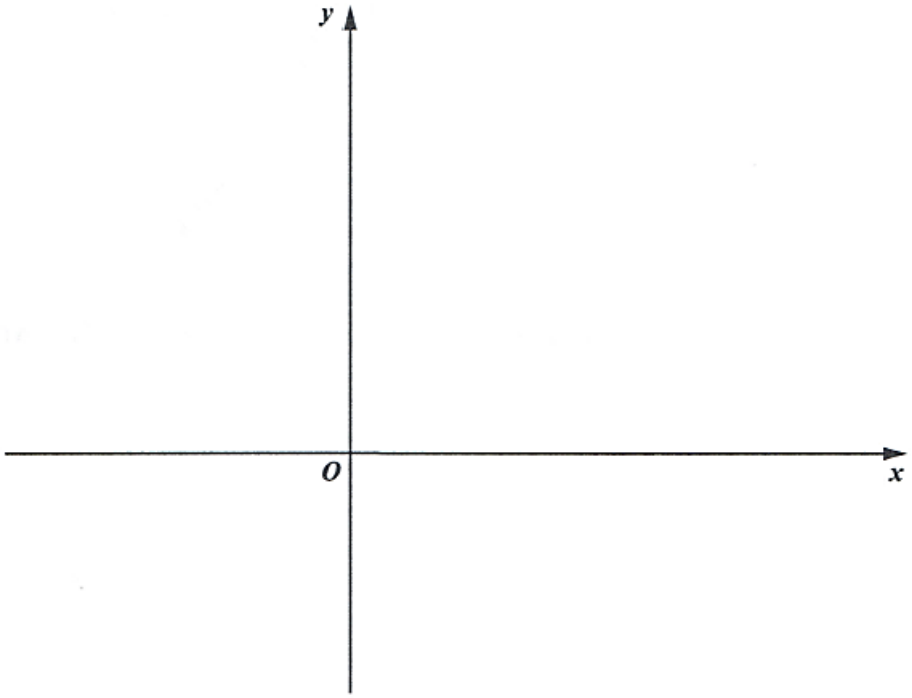
(i) equation that represents the line g

(ii) equation that represents the line h

(iii) coordinates of the point P. **Show all working.**

(b) (i) Write $4x^2 - 24x + 31$ in the form $a(x + h)^2 + k$.

(ii) On the axes below, sketch the graph of $4x^2 - 24x + 31$, indicating the coordinates of the maximum/minimum point and the y-intercept.



(3 marks)

(iii) State the equation of the axis of symmetry.

May/June 2024 #4

4. (a) A rectangle, $PQRS$, has a diagonal, PR, where P is the point $(-3, 10)$ and R is the point $(4, -4)$.

(i) Calculate the length of the line PR.

(ii) Determine the equation of the line PR.

(b) Two functions, f and g, are defined as follows.

$f(x) = 3x + 1$ and $g(x) = x^2$.

Find, in its simplest form, an expression for

(i) $f(x - 2)$

(ii) $g(3x + 2) + 10$.

May/June 2024 #8

8. (a) The functions f and g are defined as follows.

$f(x) = \frac{2x-1}{3}$ and $g(x) = 5 - x^2$.

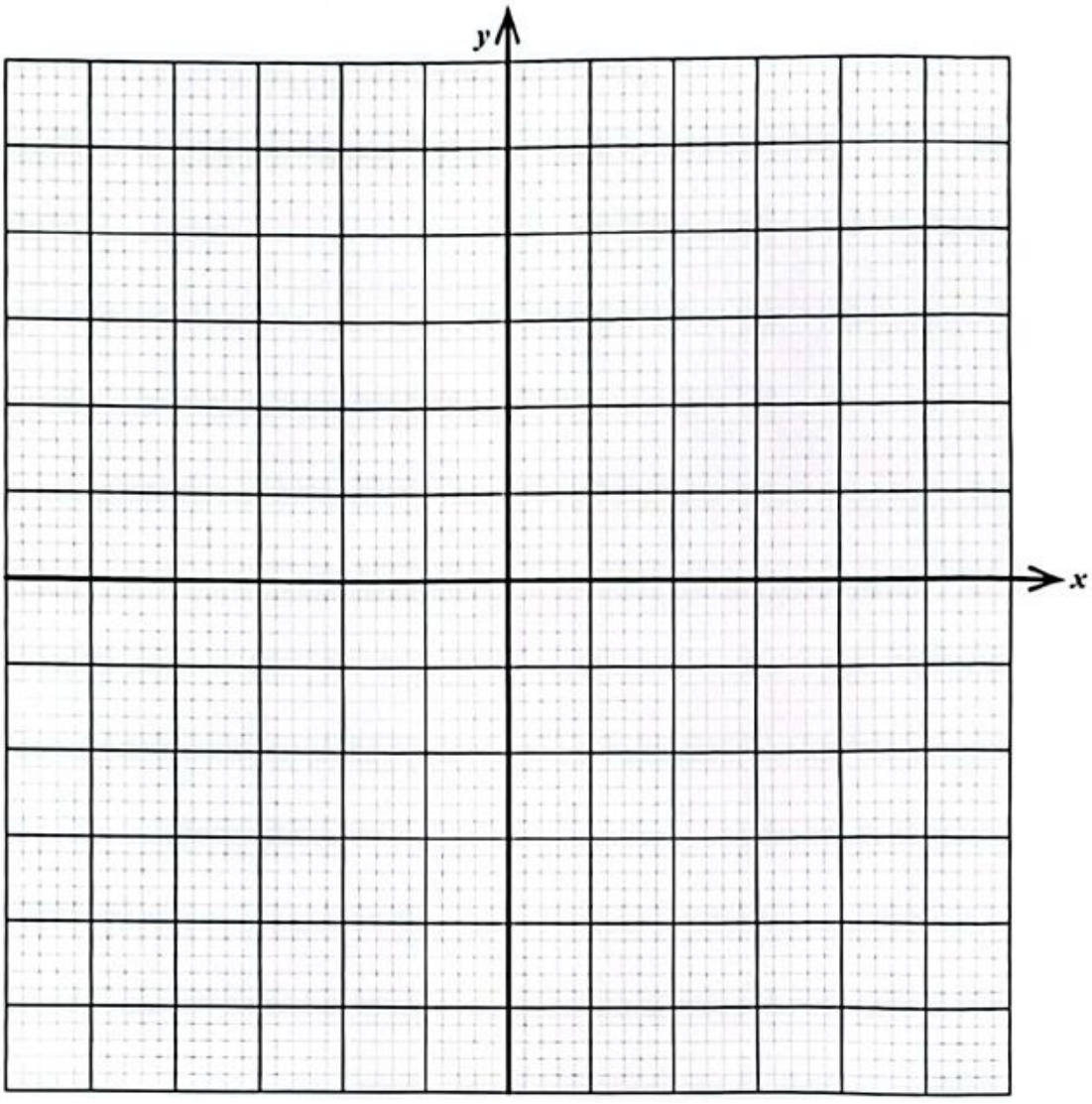
(i) Determine the value of

a) $g(2)$

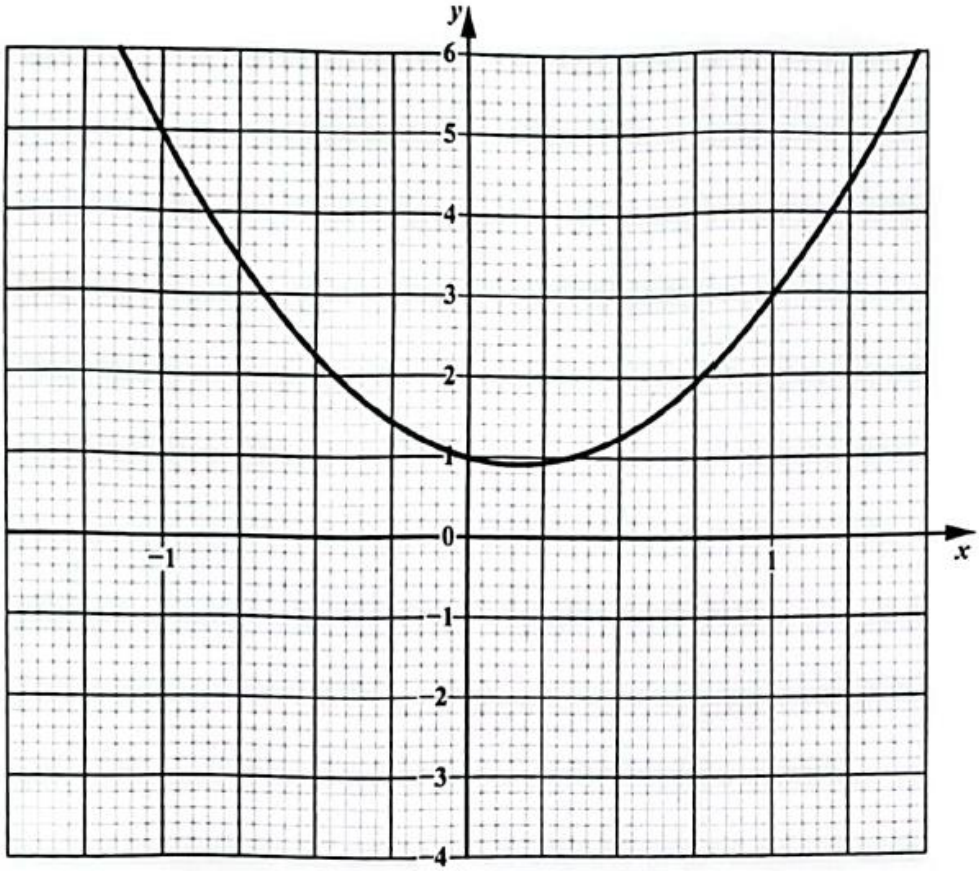
b) $f^{-1}(3)$.

(ii) Derive an expression, in its simplest form, for $fg(x)$.

(iii) Sketch the graph of the function $g(x)$ in the space provided below. On your sketch, indicate the maximum/minimum point and the roots of the function.



(b) The graph below shows a quadratic function.



(i) On the grid above, draw the tangent to the curve at $x = 1$. **(1 mark)**

(ii) Use the tangent drawn to estimate the gradient of the curve at $x = 1$.

(iii) Write down the equation of the tangent in the form $y = mx + c$.

Jan 2025 #4

4. A line segment joins the points $C\,(-5, 6)$ and $D\,(7, 2)$.

(a) Calculate the midpoint of the line segment CD.

(b) Find the gradient of the line segment CD.

(c) Determine the equation of the perpendicular bisector of CD.

(d) Another line, AB, is parallel to CD and passes through the point $(0, 1)$. Write down the equation of the line AB.

Jan 2025 #8

8. (a) The functions f and g are defined as follows

$$f: x \to \frac{1+3x}{x-1}, x \neq 1$$

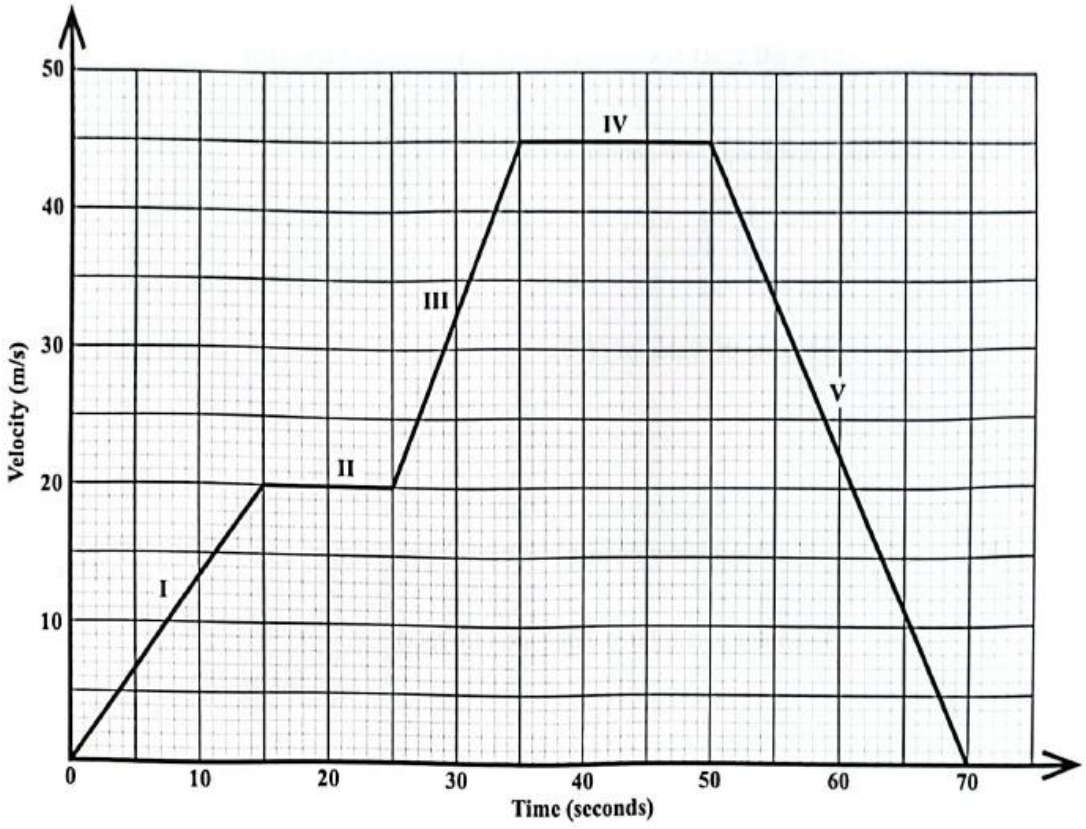
$$g: x \to 5-x.$$

(i) Calculate the value of $f(-2)$.

(ii) Determine a simplified expression for $fg(x)$.

(iii) Derive an expression in terms of x for the inverse function, $f^{-1}(x)$.

(b) The velocity–time graph below describes the journey of a car over a period of 70 seconds. The journey is represented in 5 stages labelled I to V.



(i) Complete the following statement.

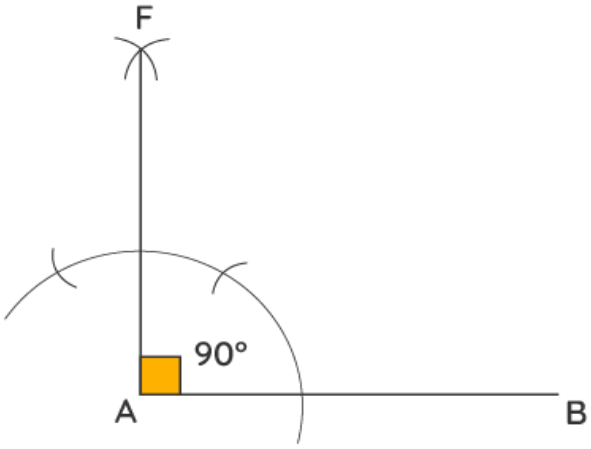
During Stage IV, the car is travelling at ..... m/s with an acceleration of ..... m/s². **(2 marks)**

(ii) Determine the MAXIMUM acceleration of the car during the 70 seconds.

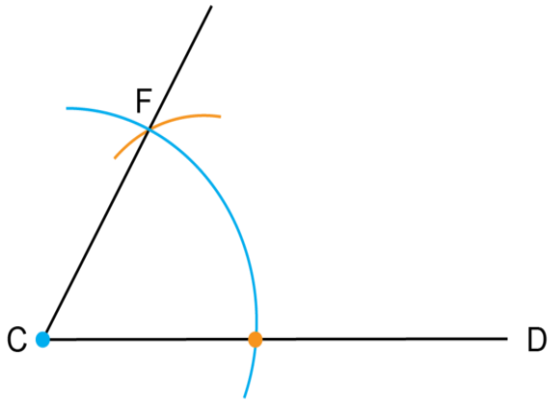
(iii) Calculate the distance travelled by the car during the **first 25 seconds** of its journey.

Section 8 – Geometry and Trigonometry

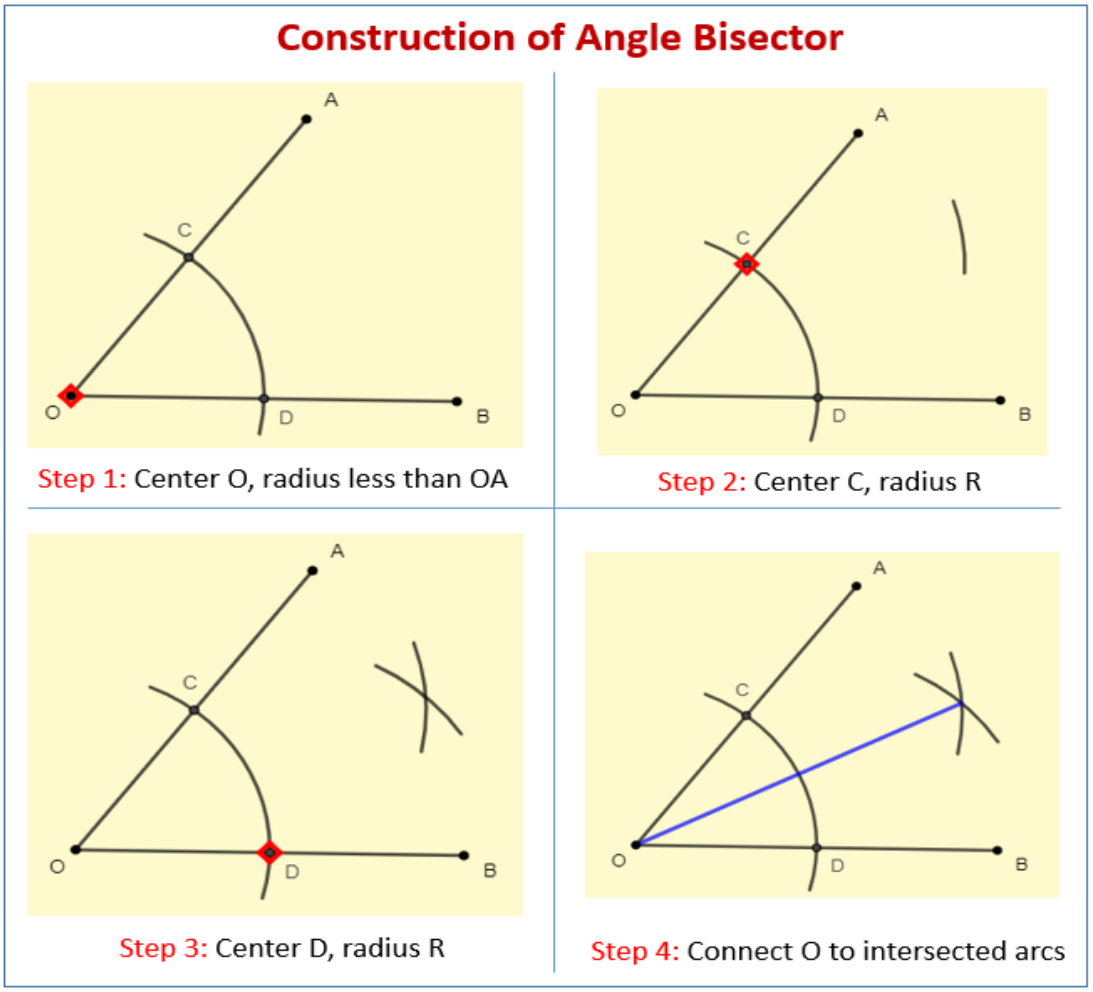
Construction:



Construction of a 90 ° angle



Construction of a 60 ° angle

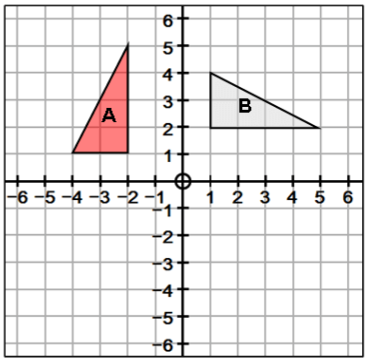


Transformations:

Describing Rotations

State...

1. The centre of rotation
2. The angle of rotation
3. The direction of rotation

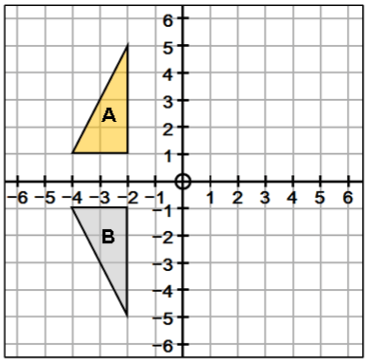


- Centre of rotation is (0,0)
- Angle of rotation is 90°
- Direction of rotation is clockwise

Describing Reflections

State...

1. The line of symmetry



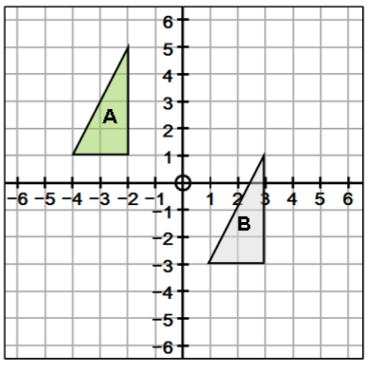
- The line of symmetry is the X axis

Describing Translations

State...

1. Movement left or right
2. Movement up or down

- Or write the column vector

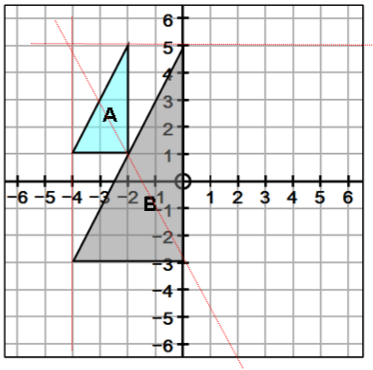


- Translation of 5 to the right and 4 down
- $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

Describing Enlargements

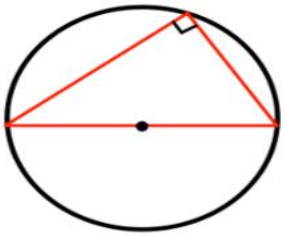
State...

1. Centre of enlargement
2. Scale Factor

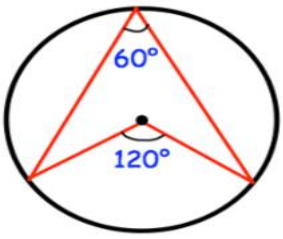


- The centre of enlargement is (-4, 5)
- The scale factor is 2

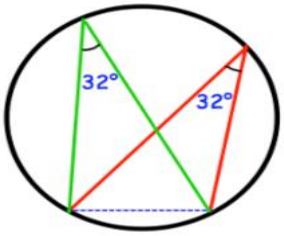
Circle Theorem:



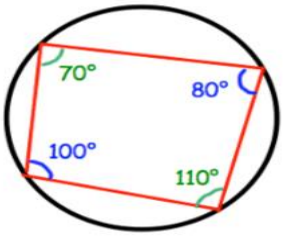
The angle in a semi-circle is 90°



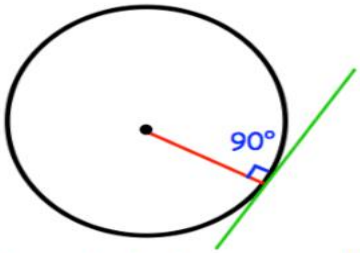
The angle at the circumference is half the angle at the centre



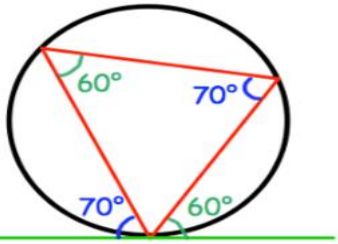
The angles in the same segment from a common chord are equal



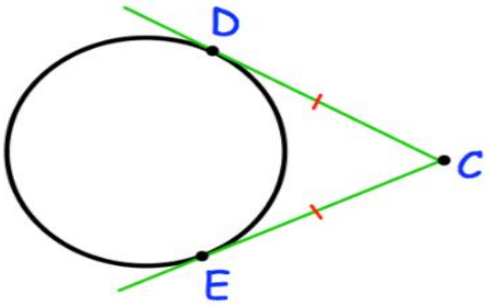
The opposite angles in a cyclic quadrilateral always add to 180°



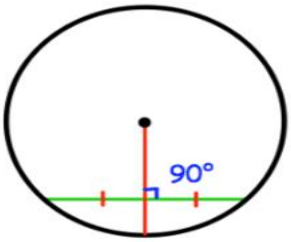
The angle between a radius and a tangent is 90°



Alternate segment theorem
The angle between the chord and the tangent is equal to opposite angle inside the triangle.

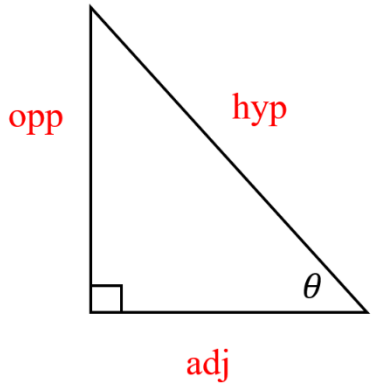


The tangents to a circle from the same point will be equal length



The radius through the midpoint of a chord will bisect the chord at 90°

Trigonometry:

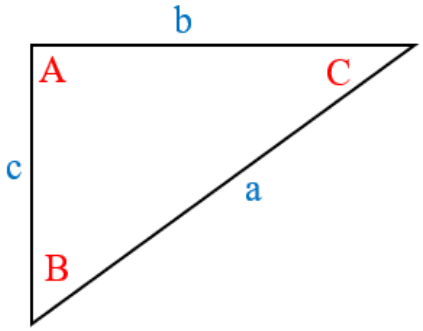


For right-angled triangles:

$$hyp^2 = opp^2 + adj^2$$

- $\sin\theta = \frac{opp}{hyp}$
- $\cos\theta = \frac{adj}{hyp}$
- $\tan\theta = \frac{opp}{adj}$

N.B. Area of triangle = $\frac{1}{2}\, b \times h$



- Sine rule: **more angles than lengths**

  $\frac{a}{\sin A} = \frac{b}{\sin B}$

  Area $= \frac{1}{2}ab\ \sin C$

- Cosine rule: **more lengths than angles**

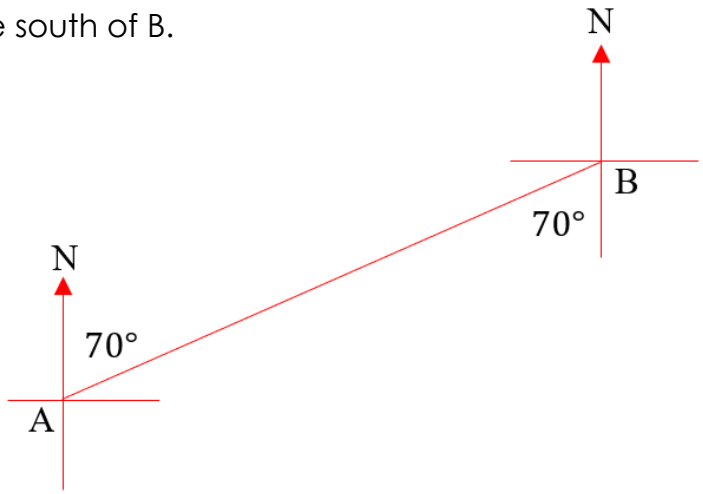
  $a^2 = b^2 + c^2 - 2bc\ \cos A$

Bearings:

All bearings are measured from a North Line, N, in a clockwise direction. All angles must be shown.

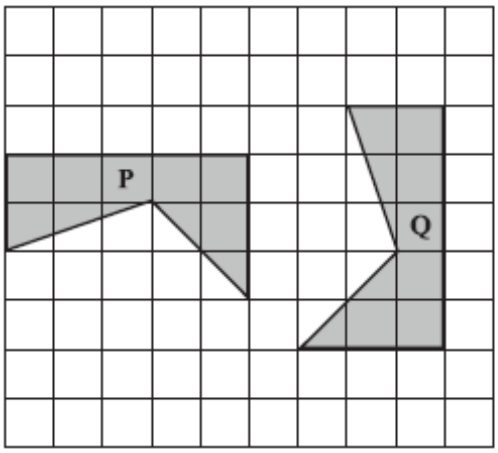
Example: B is on a bearing of 70° from A

C is due south of B.



Jan 2021 #3

3. (a) The diagram below shows two pentagons, P and Q, drawn on a grid made up of squares.



(i) Select the correct word from the following list to complete the statement below.

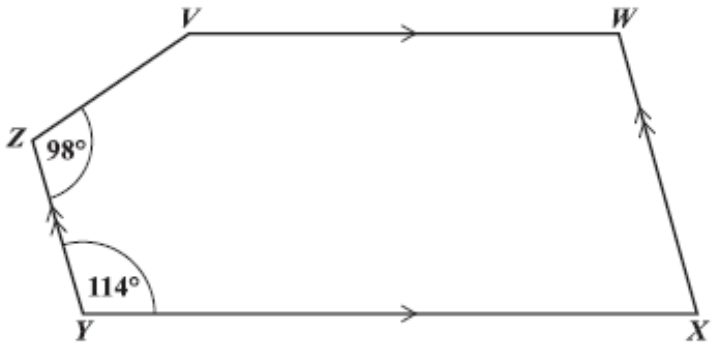
opposite	reflected	congruent	translated

Pentagon P is ..... to Pentagon Q.

(1 mark)

(ii) Give the reason for your choice in (a) (i).

(b) The diagram below, **not drawn to scale**, shows the pentagon $VWXYZ$. In the pentagon, YZ is parallel to XW and YX is parallel to VW. Angle $XYZ = 114°$ while angle $VZY = 98°$.

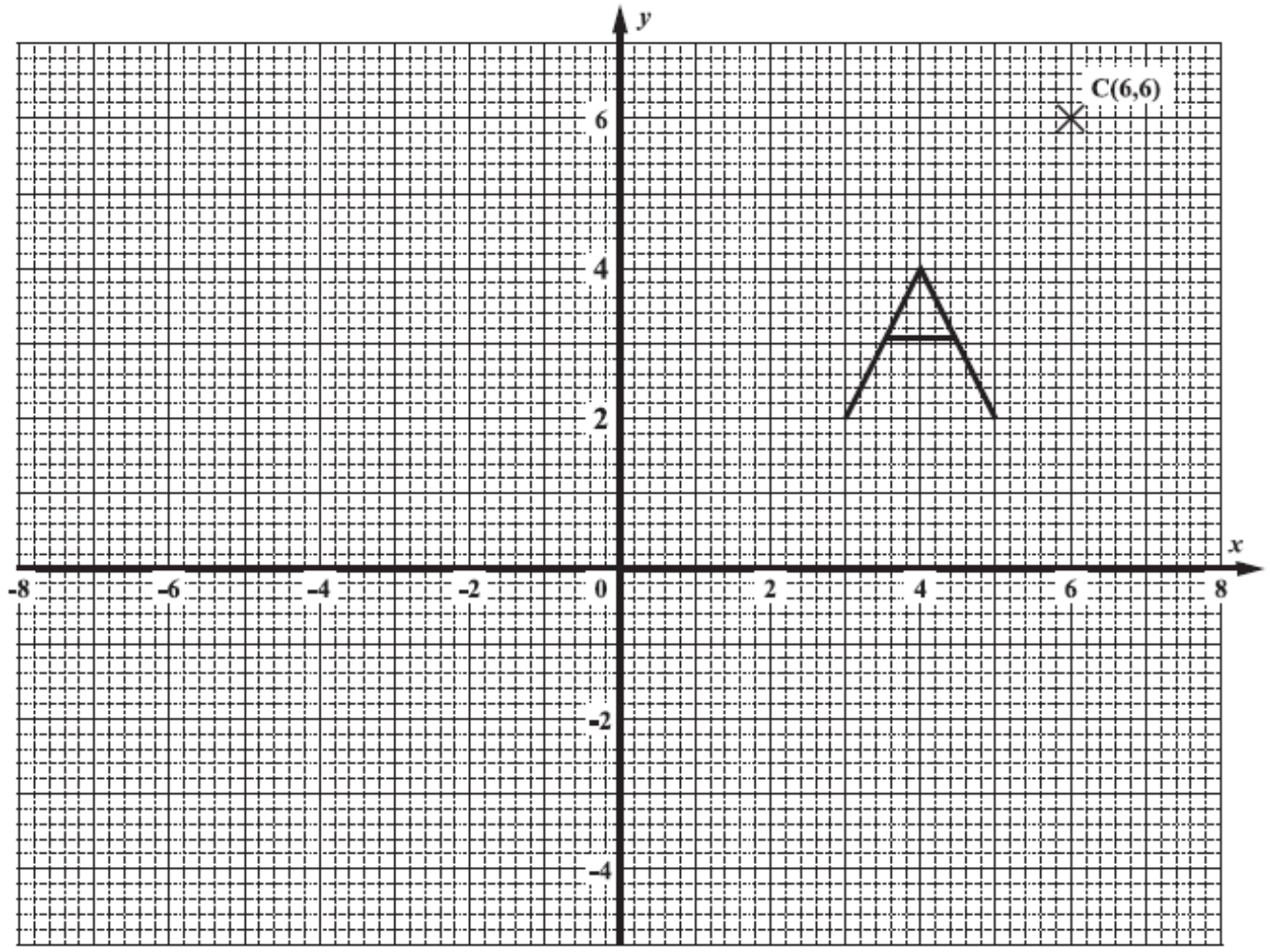


Determine the value of

(i) angle WXY

(ii) angle ZVW.

(c) The letter 'A' and a point C(6, 6) are shown on the grid below.



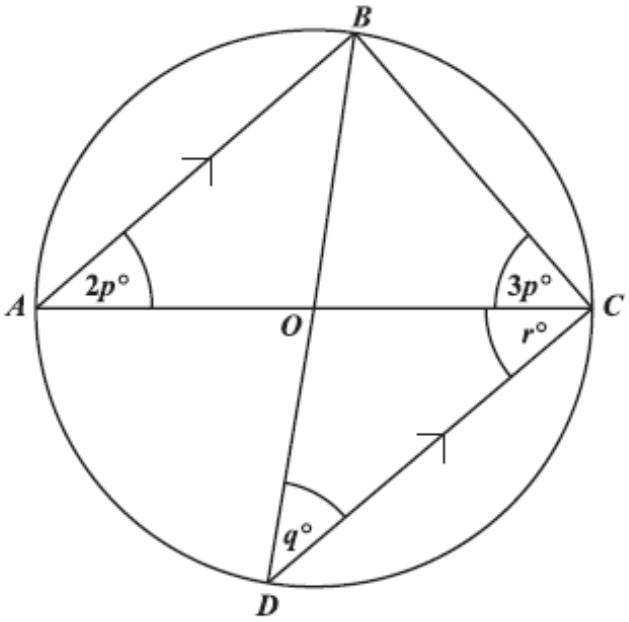
On the diagram, draw accurately, EACH of the following transformations.

(i) The enlargement of letter 'A' by scale factor 2, about centre, C(6, 6). **(2 marks)**

(ii) The translation of letter 'A' using the vector $T = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$. **(2 marks)**

Jan 2021 #9

9. (a) In the diagram below, A, B, C and D are points on the circumference of a circle, with centre O. AOC and BOD are diameters of the circle. AB and DC are parallel.



(i) State the reason why angle ABC is 90°.

(ii) Determine the value of EACH of the following angles. Show detailed working where necessary and give a reason to support your answer.

a) Angle BAC

Reason

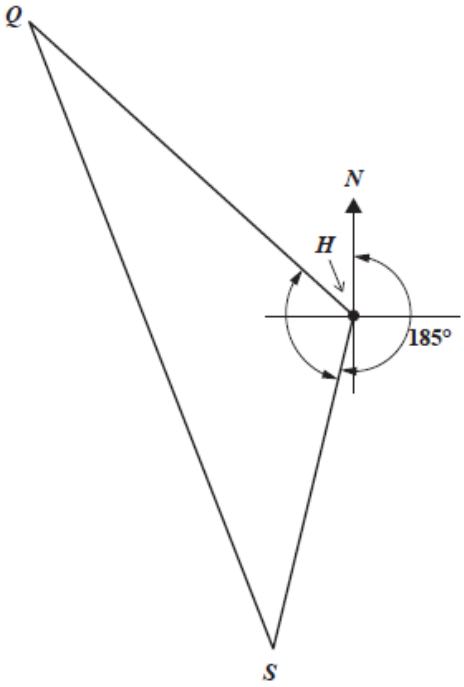
b) Angle q

Reason

(iii) Calculate the value of angle r.

(b) From a harbour, H, the bearing of two buoys, S and Q, are 185° and 311° respectively. Q is 5.4 km from H while S is 3.5 km from H.

(i) On the diagram below, which shows the sketch of this information, insert the value of the marked angle, QHS. **(1 mark)**

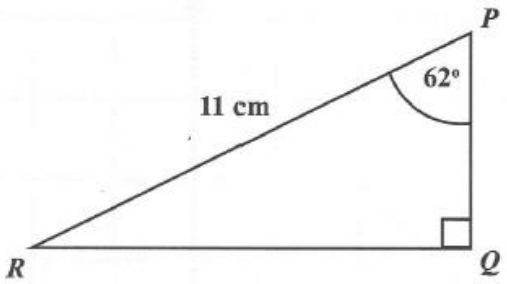


(ii) Calculate QS, the distance between the two buoys.

(iii) Calculate the bearing of S from Q.

May/June 2021 #3

3. (a) The diagram below shows the triangle PQR in which angle $QPR = 62°$, angle $PQR = 90°$ and $PR = 11$ cm.

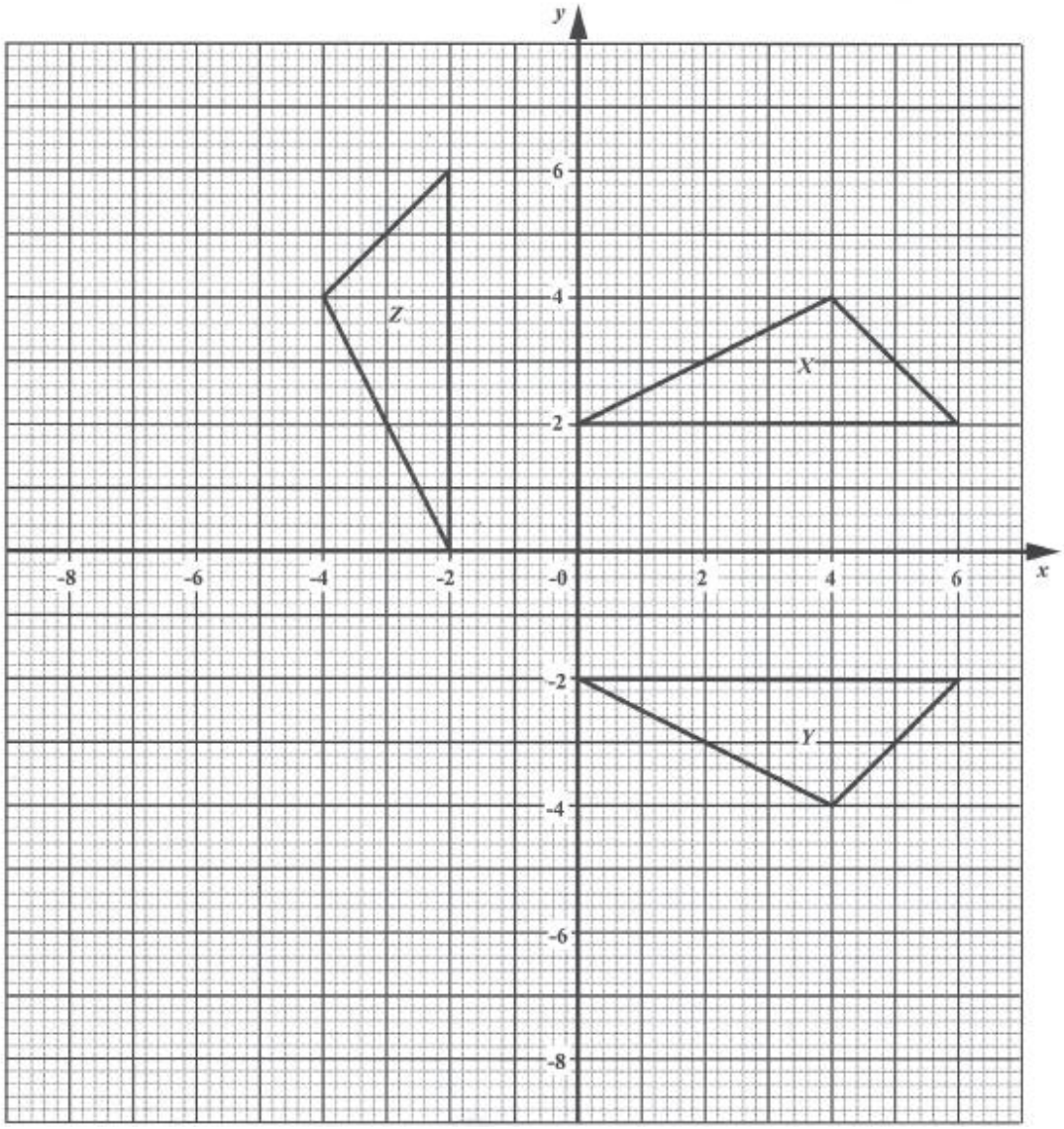


Calculate

(i) the size of angle PRQ

(ii) the length of the side RQ.

(b) The diagram below shows three triangles, X, Y and Z, on a square grid.



(i) Triangle X is mapped onto Triangle Y by a reflection. State the equation of the mirror line.

(ii) Describe fully the transformation which maps Triangle X onto Triangle Z.

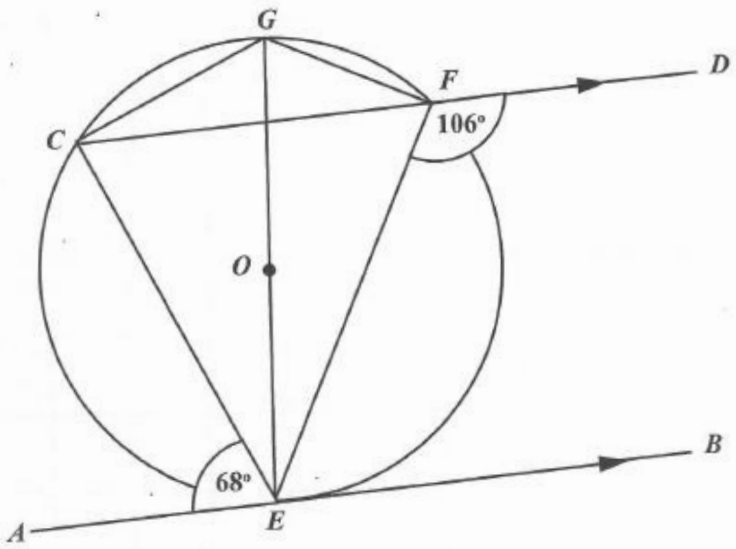
(iii) On the diagram **on page 10**, translate Triangle Y using the vector $\begin{bmatrix} -7 \\ 1 \end{bmatrix}$.

Label this image V. **(1 mark)**

(iv) On the diagram **on page 10**, enlarge Triangle X about the centre, $C(0, 0)$, and scale factor $\frac{1}{2}$. Label this image W. **(2 marks)**

May/June 2021 #9

9. (a) In the diagram below, E, C, G and F are points on the circumference of a circle. EG is a diameter of the circle. The tangent AEB is parallel to CD. Angle $AEC = 68°$ and angle $EFD = 106°$.



Determine the value of EACH of the following angles. Show detailed working where necessary and give a reason to support your answer.

(i) ECD

Reason

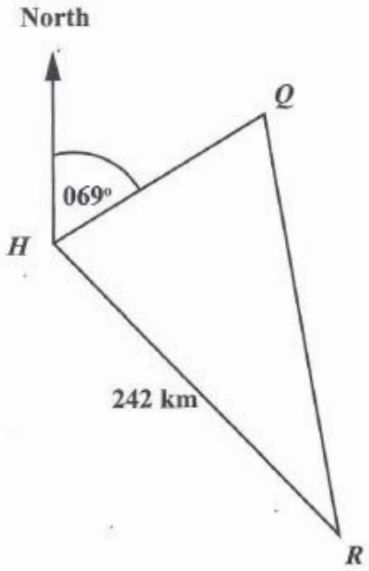
(ii) CEG

Reason

(iii) CGF

Reason

(b) From a harbour, H, the bearing of two ships, Q and R, are 069° and 151° respectively. Q is 175 km from H while R is 242 km from H.



(i) Complete the diagram above to show the information given. **(1 mark)**

(ii) Calculate QR, the distance between the two ships, to the nearest km.

(iii) Calculate how far due south is Ship R of the harbour, H.

Jan 2022 #3

3. (a) **The box below contains the names of 5 quadrilaterals.**

Trapezium	Rhombus	
Kite	Square	Rectangle

Choose the name of one quadrilateral from the box that BEST completes each statement.

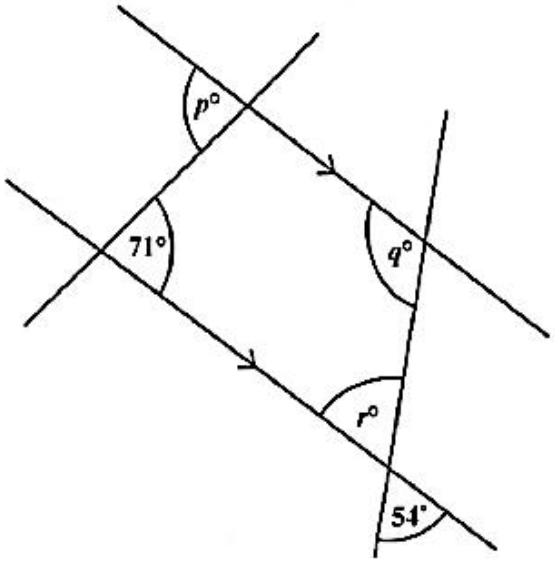
(i) A ..... **has no lines of symmetry and has rotational symmetry of order one.**

(ii) A ..... **has EXACTLY two lines of symmetry and 4 right angles.**

(iii) A ..... **has one line of symmetry but no rotational symmetry.**

(3 marks)

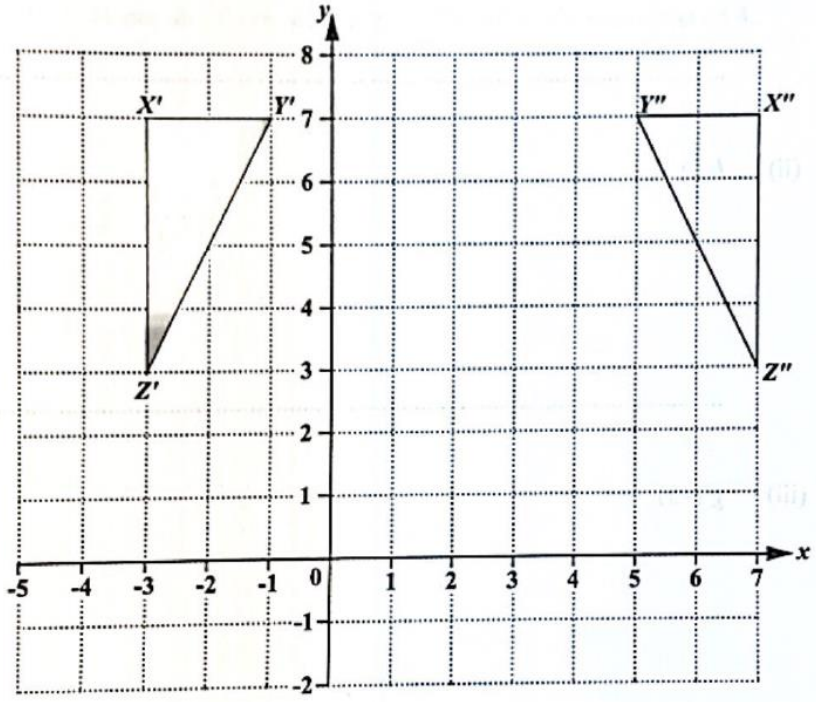
(b) The diagram below shows 4 straight lines, 2 of which are parallel.



(i) Determine the values of q and r.

(ii) Give a geometrical reason why $\angle p = 71°$.

(c) The diagram below shows triangles $X'Y'Z'$ and $X'''Y'''Z''$ drawn on a square grid.



(i) Triangle $X'Y'Z'$ is the image of Triangle XYZ after an enlargement of scale factor 2, with centre (5, 1).

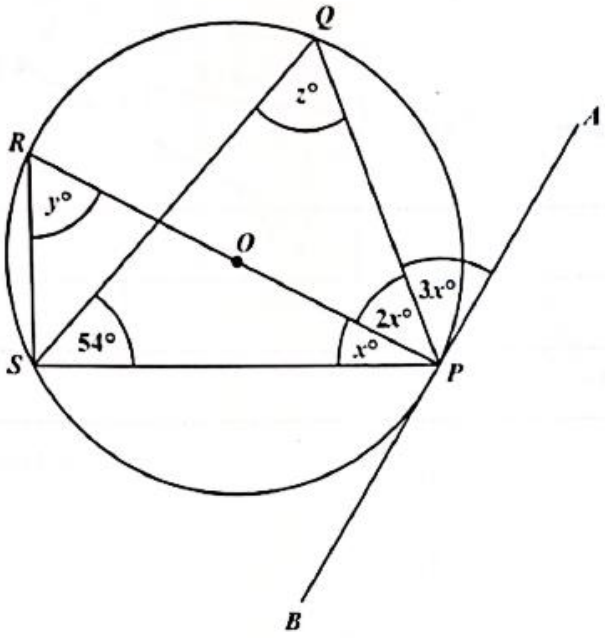
Draw triangle XYZ, the **OBJECT** for Triangle $X'Y'Z'$, on the grid above.

(2 marks)

(ii) Triangle $X'Y'Z'$ is mapped onto Triangle $X''Y''Z''$ by a reflection in the line P. State the equation of the mirror line, P.

Jan 2022 #9

9. (a) The diagram below shows a circle, with the points P, Q, R and S lying on its circumference and its centre marked O. RP is a diameter of the circle and AB is a tangent to the circle at P. Angle $APQ = 3x°$, angle $QPR = 2x°$, angle $RPS = x°$ and angle $QSP = 54°$.



Determine the value of EACH of the following angles. Show detailed working where possible and give a **reason** for your answer.

(i) x

Reason

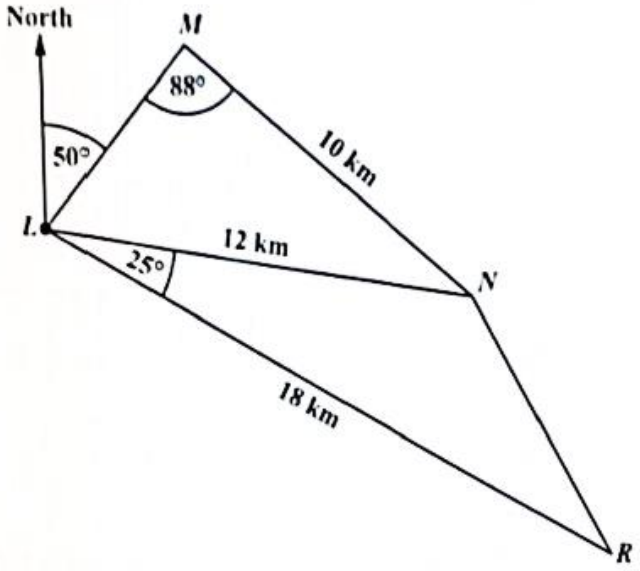
(ii) y

Reason

(ii) z

Reason

(b) The diagram below shows straight roads connecting the towns L, M, N and R. $LR = 18$ km, $LN = 12$ km and $MN = 10$ km. Angle $RLN = 25°$ and angle $LMN = 88°$.



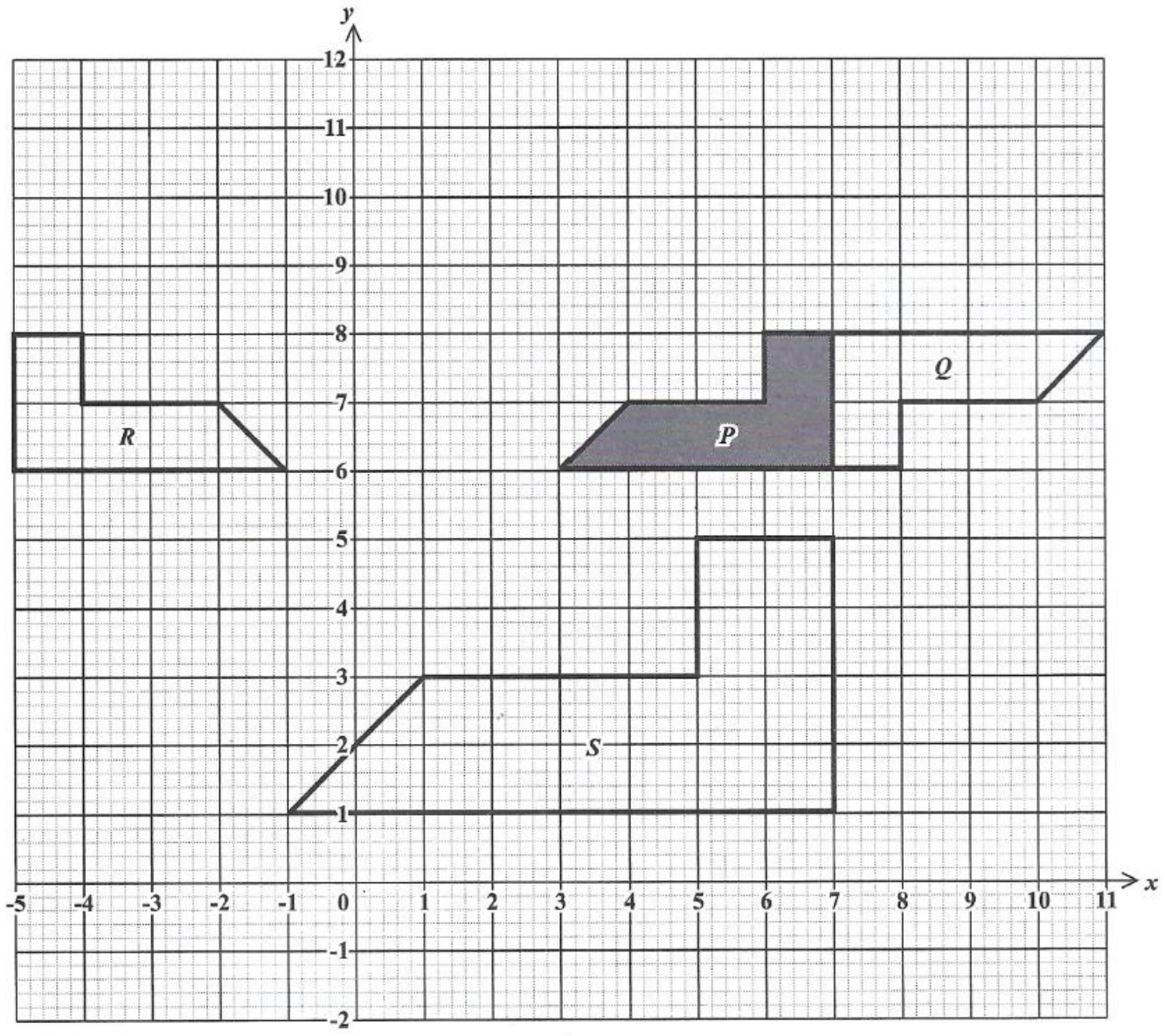
(i) Calculate angle MLN.

(ii) Calculate the distance NR.

(iii) Determine the bearing of Town R from Town L.

May/June 2022 #3

3. The diagram below shows four shapes, P, Q, R and S, on a square grid.



(a) Describe fully the **single** transformation that maps shape P onto shape

(i) Q

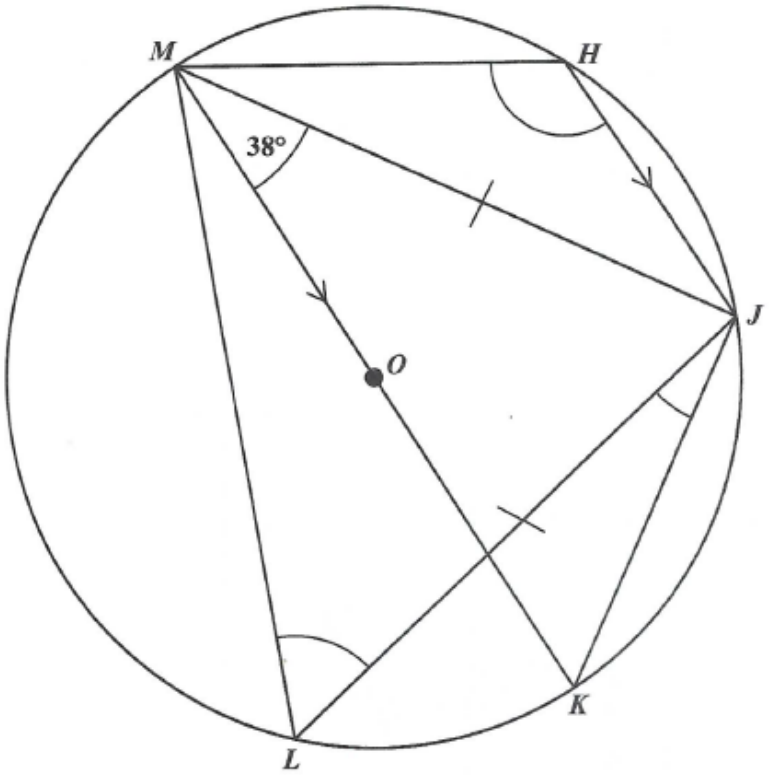
(ii) R

(iii) S.

(b) On the grid provided **on page 10**, draw the image of shape P after a translation by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$. Label this image T. **(1 mark)**

May/June 2022 #9

9. (a) H, J, K, L and M are points on the circumference of a circle with centre O. MK is a diameter of the circle and it is parallel to HJ. $MJ = JL$ and angle $JMK = 38°$.



(i) Explain, giving a reason, why angle

a) $HJM = 38°$

b) $MJK = 90°$.

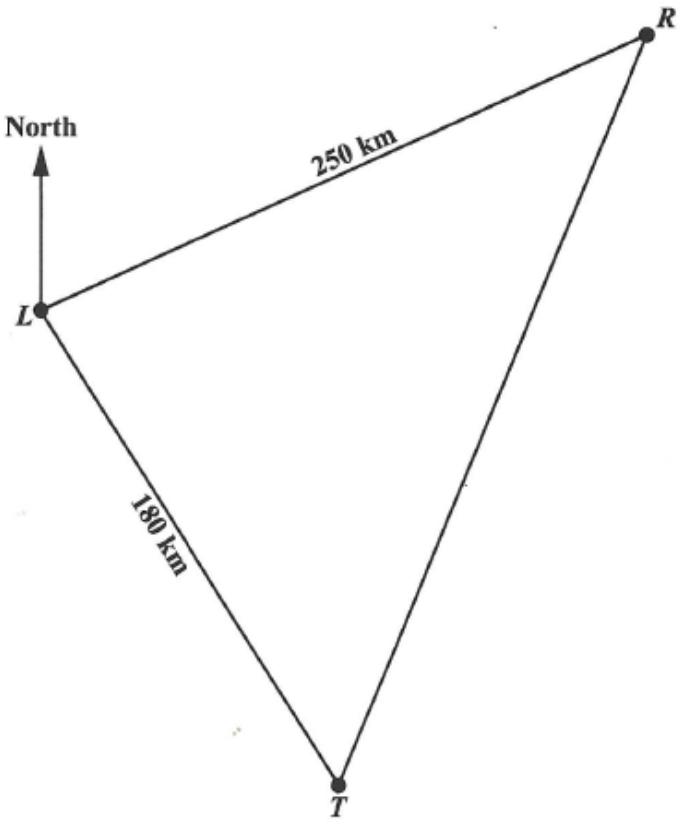
(ii) Determine the value of EACH of the following angles. Show detailed working where appropriate.

a) Angle MLJ

b) Angle LJK

c) Angle JHM

(b) From a port, L, ship R is 250 kilometres on a bearing of 065°. Ship T is 180 kilometres from L on a bearing of 148°. This information is illustrated in the diagram below.



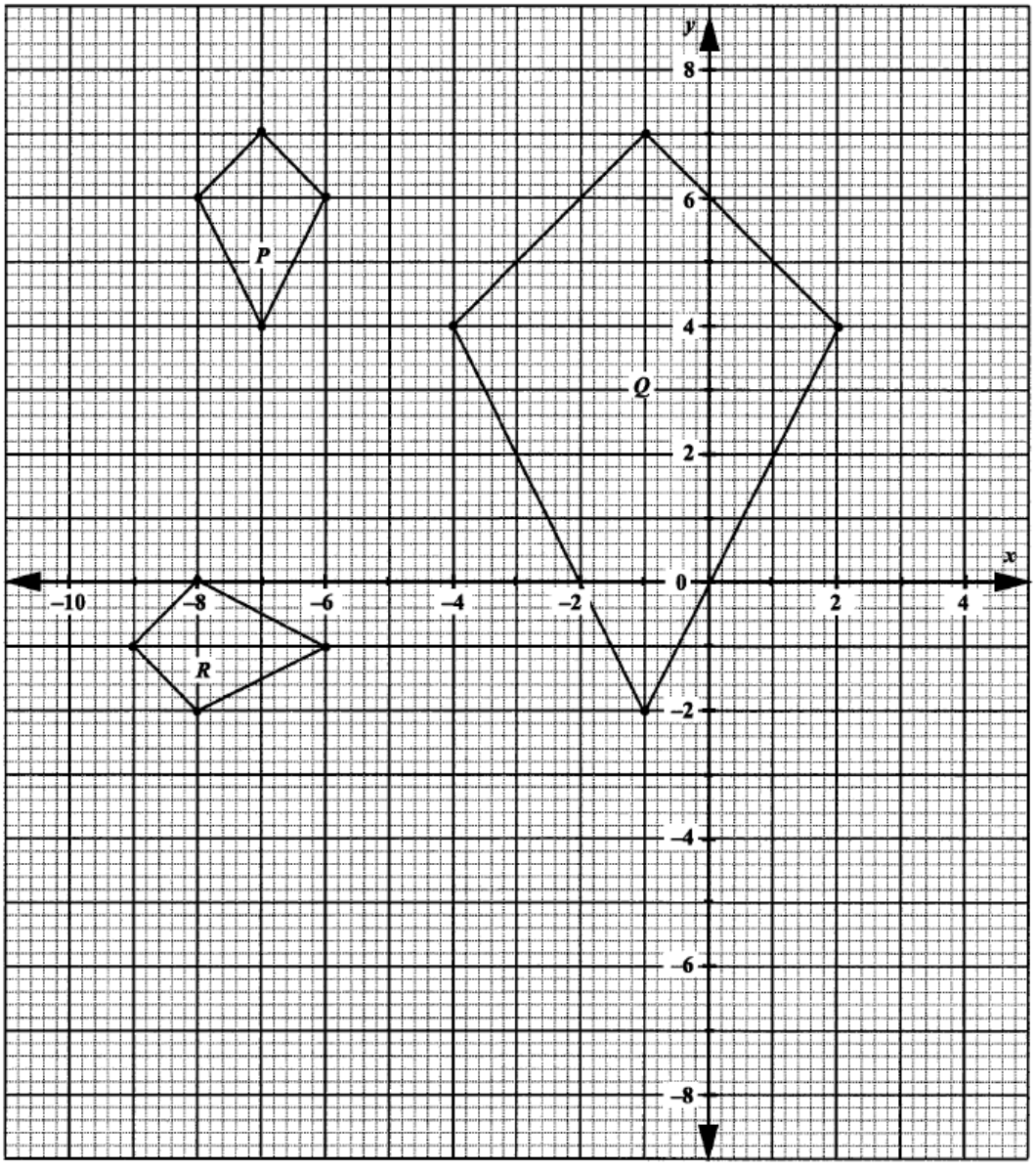
(i) Complete the diagram above by inserting the value of angle RLT. **(1 mark)**

(ii) Calculate RT, the distance between the two ships.

(iii) Determine the bearing of T from R.

Jan 2023 #3

3. The following diagram shows 3 quadrilaterals, P, Q and R on a square grid. Q and R are the images of P after it underwent 2 different transformations.



(a) On the grid **on page 9**, draw the image of quadrilateral P after a

(i) translation by the vector $\begin{pmatrix} 10 \\ -4 \end{pmatrix}$. Label this image P' **(1 mark)**

(ii) reflection in the line $y = 0$. Label this image P''. **(2 marks)**

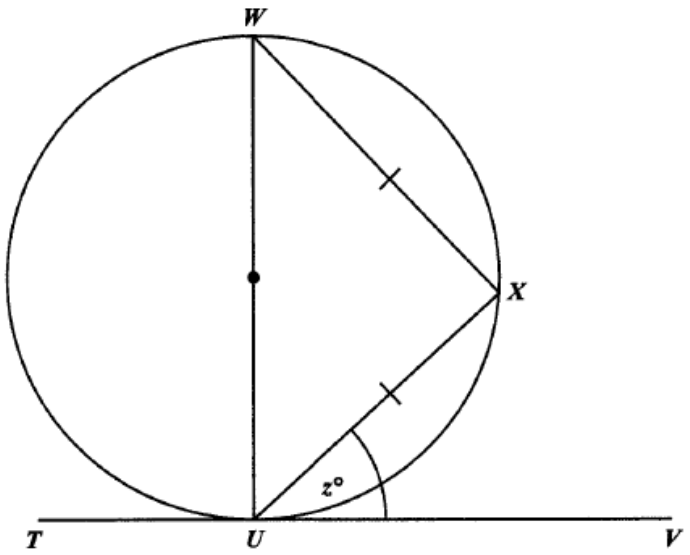
(b) Describe fully a **single** transformation that maps Quadrilateral P onto

(i) Quadrilateral Q

(ii) Quadrilateral R.

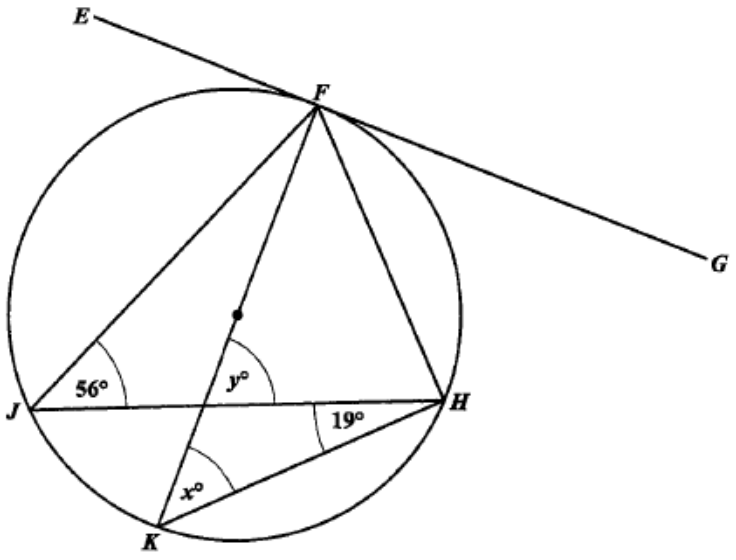
Jan 2023 #9

9. (a) *W, X* and *U* are points on the circumference of a circle. *TV* is a tangent to the circle at *U*. *UW* is a diameter of the circle and triangle *WXU* is isosceles.



Using appropriate theorems, state THREE reasons that explain why the measure of Angle *z* is 45°.

(b) The diagram below shows a circle with diameter *KF*. Line *EFG* is a tangent to the circle at *F*. The points *F, H, K* and *J* lie on the circumference of the circle.

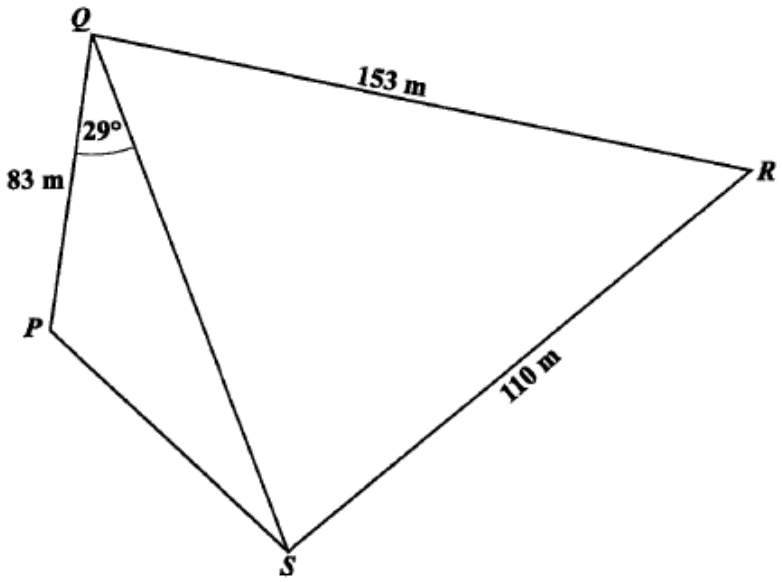


By showing EACH step in your work, where appropriate, find the value for EACH of the following angles:

(i) Angle *x*

(ii) Angle *y*.

(c) The diagram below shows 4 points, P, Q, R and S on level ground, where pillars will be placed to mark the outline for a foundation.

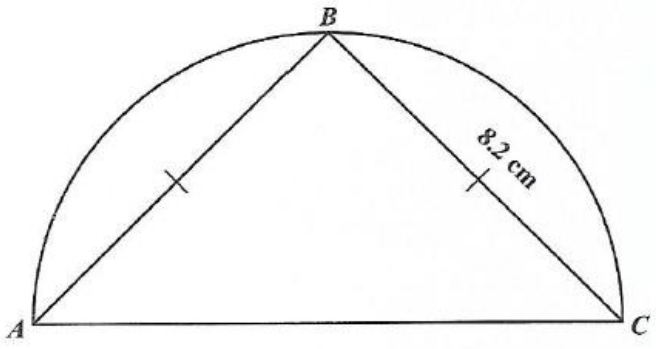


(i) There is a vertical post, RT, at R. From Q, the angle of elevation of the top of the post, T, is 21°. Find the height of the post.

(ii) Given that the length QS is 135 m, calculate the perimeter of the foundation $PQRS$.

May/June 2023 #3

3. (a) The diagram below shows a semicircle with diameter AC. B is a point on the circumference and $AB = BC = 8.2$ cm.

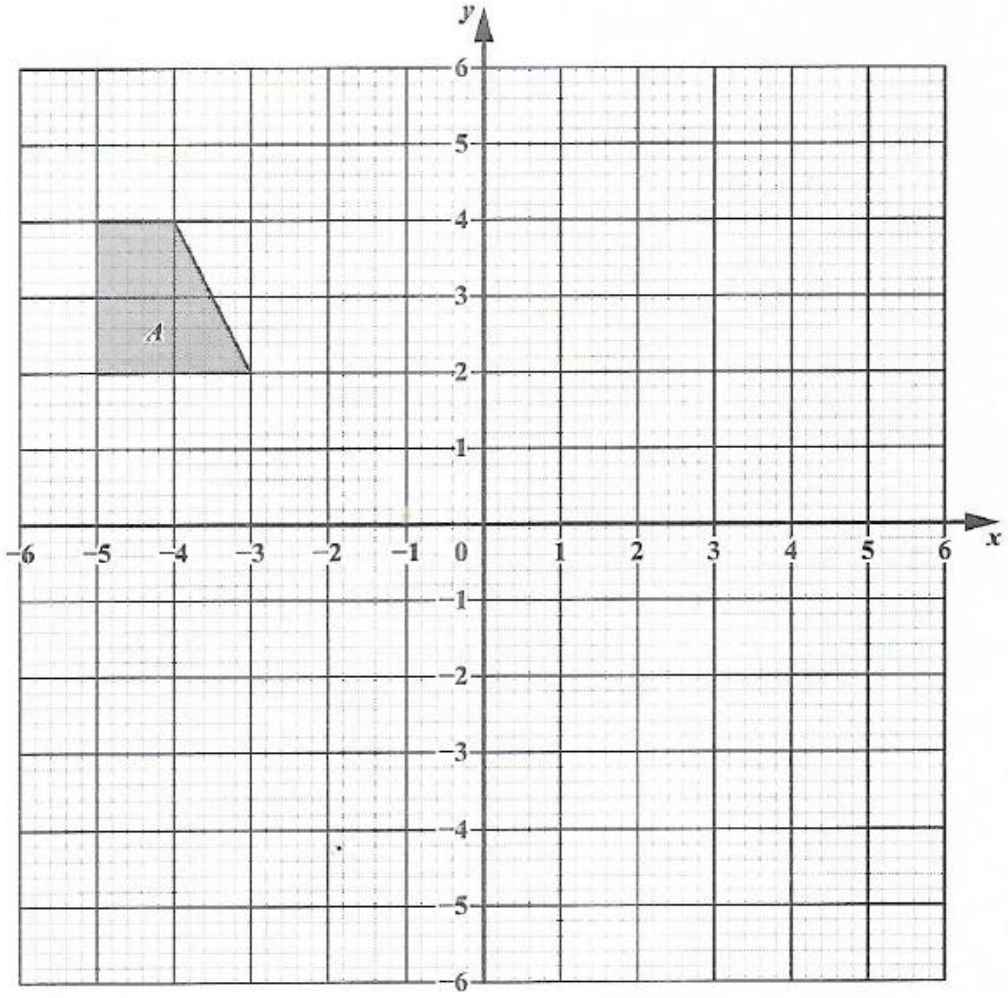


(i) State the geometrical name of the line AB.

(ii) Find the value of the radius of the circle.

(b) Each interior angle of a regular polygon is 160°. Calculate the number of sides of the polygon.

(c) The diagram below shows a trapezium, A, drawn on a square grid.



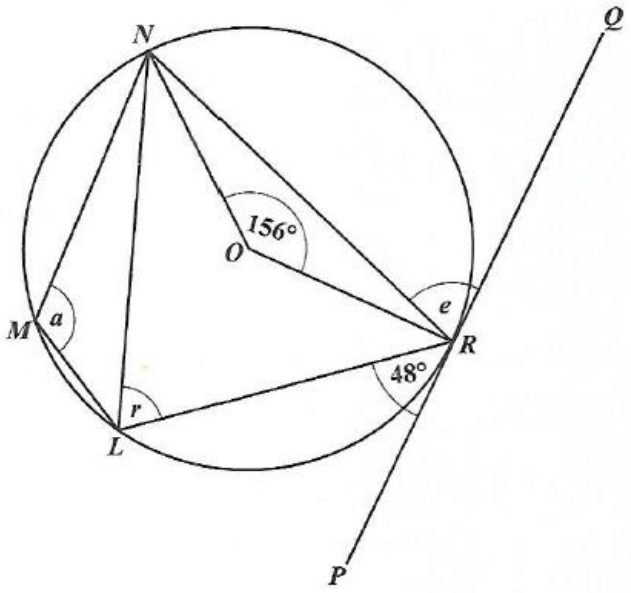
On the diagram above, draw the image of A after it undergoes a

(i) reflection in the line $x = -1$ and label this image A'. **(2 marks)**

(ii) translation with vector $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ and label this image A''. **(1 mark)**

May/June 2023 #9

9. (a) L, M, N and R are points on the circumference of a circle, with centre O. PQ is a tangent to the circle at R. Angle $PRL = 48°$ and Angle $RON = 156°$.



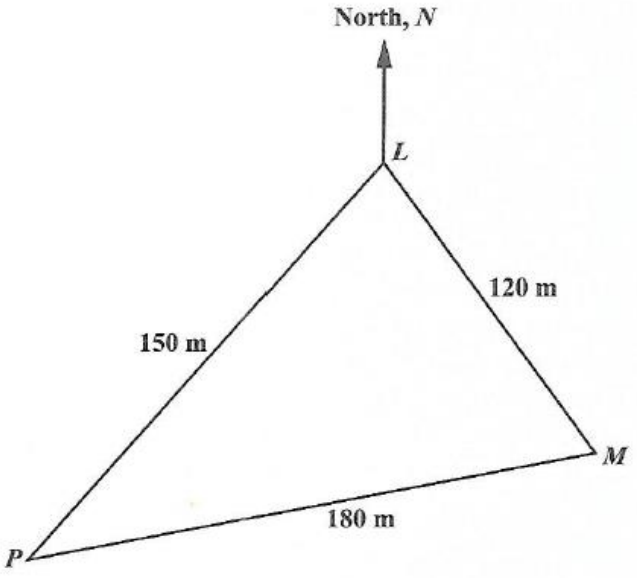
Find the value of EACH of the following angles, giving reasons for EACH of your answers. Show ALL working where appropriate.

(i) Angle r

(ii) Angle e

(iii) Angle a

(b) The diagram below shows a triangular field, LMP, on horizontal ground.



(i) Calculate the value of Angle MLP.

(ii) The bearing of P from L is 210°.

a) Find the bearing of M from L.

b) Calculate the value of Angle NLP and hence, find the bearing of L from P.

Jan 2024 #3

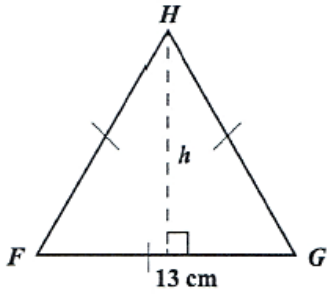
3. (a) In triangle ABC, $AC = 8$ cm and $BC = 5$ cm.

(i) Using a ruler and compasses only, construct triangle ABC. The line AB has been drawn for you.

(2 marks)

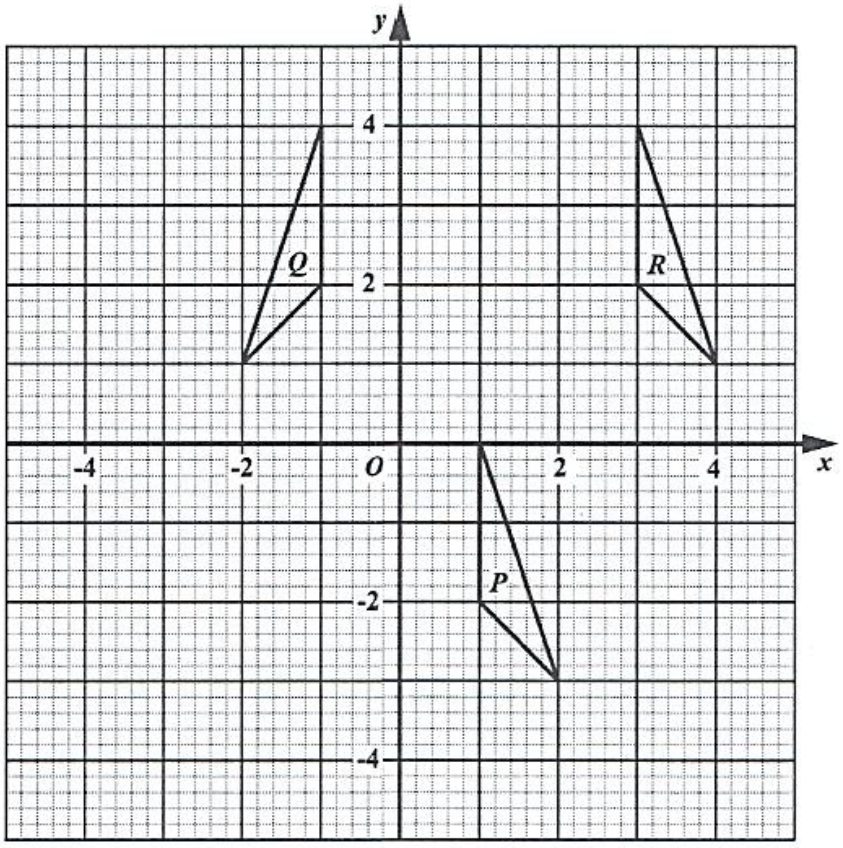
(ii) Measure and state the value of Angle BAC.

(b) The diagram below shows an equilateral triangle, FGH, whose base is 13 cm and its height, h.



Calculate the value of h.

(c) The diagram below shows 3 triangles, P, Q and R. Triangles Q and R are images of Triangle P after it undergoes a double transformation.



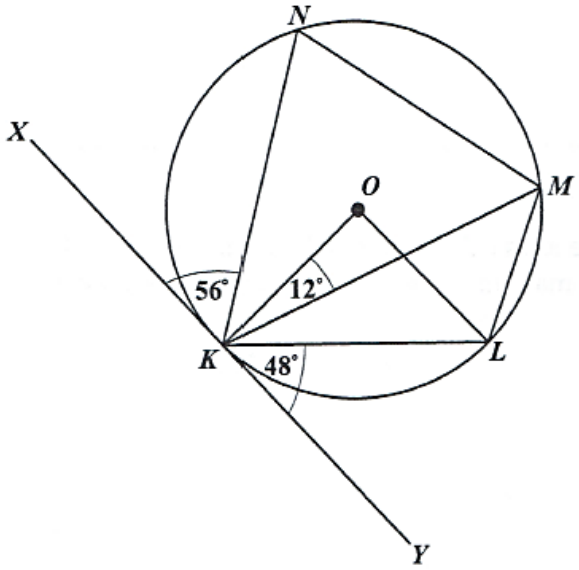
Describe fully the single transformation that maps Triangle

(i) P onto Triangle R

(ii) R onto Triangle Q.

Jan 2024 #9

9. (a) *K*, *L*, *M* and *N* are points on the circumference of a circle with centre *O*. *XY* is a tangent to the circle at *K*. Angle *LKY* = 48°, *MKO* = 12° and Angle *XKN* = 56°.



(i) Find the value of Angle *KOL*, giving reasons for EACH step of your work.

(ii) Find the value of EACH of the following angles.

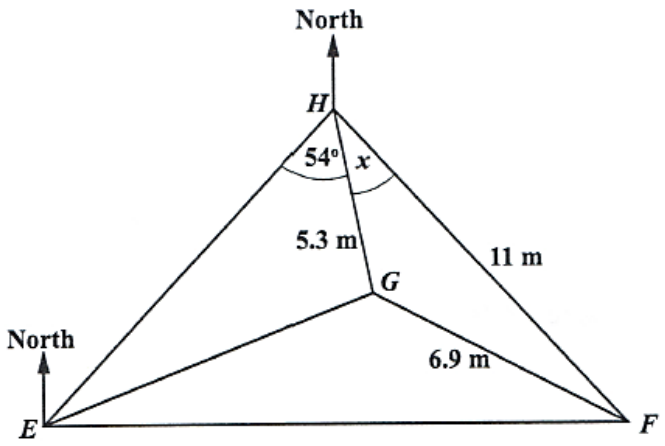
a) Angle *LMN*

b) Angle *KLO*

c) Angle *MLK*

d) Angle *KNM*

(b) *E, F, G* and *H* are 4 points on level ground. The diagram below gives information on the distances and angles between the points.



(i) Show that the value of x is 29.5°, correct to 1 decimal place.

(ii) A vertical tower, *GT*, is constructed at the point *G* and is pivoted to the ground at the points *E*, *F* and *H* using pieces of wire. The angle of elevation of the top of the tower, *T*, from the point *F* is 31°.

What length of wire was used to secure Point *T* to Point *F*?

(iii) The bearing of *E* from *H* is 228°. Find the bearing of

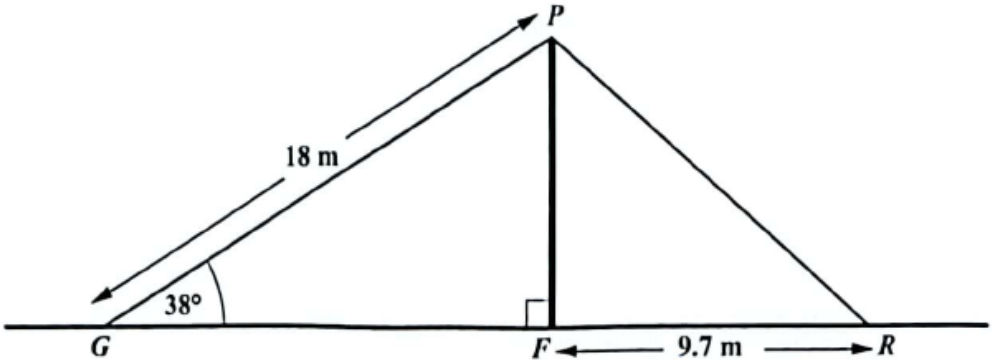
a) *H* from *E*

b) *G* from *H*.

May/June 2024 #3

3. (a) A vertical flagpole, FP, stands on horizontal ground and is held by two ropes, PG and PR, as shown in the diagram below.

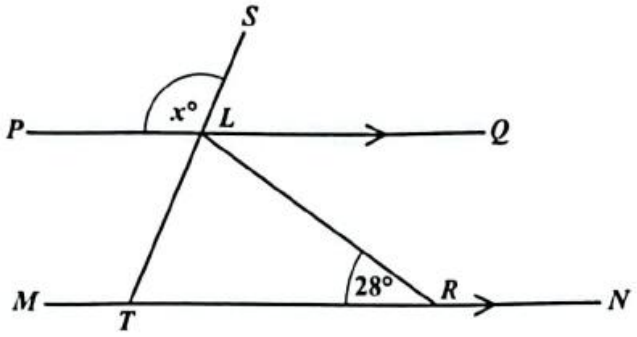
$PG = 18$ m, $FR = 9.7$ m and angle $FGP = 38°$.



(i) Calculate the height of the flagpole, FP.

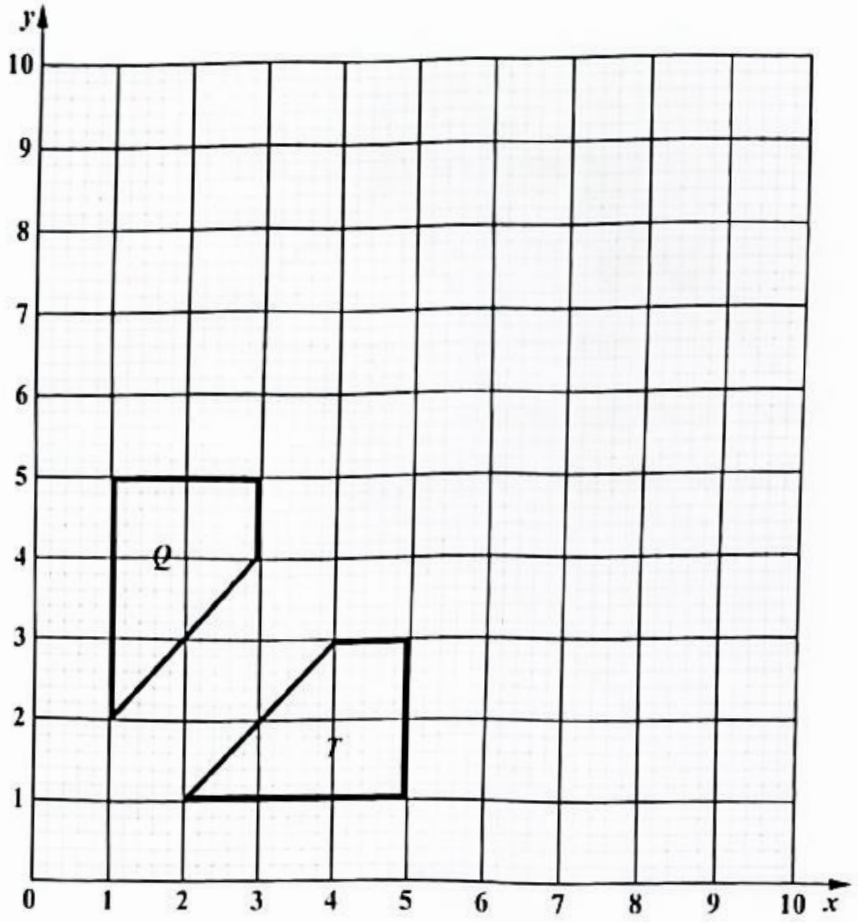
(ii) Find PR, the length of one of the pieces of rope used to hold the flagpole.

(b) In the diagram below, PQ is parallel to MN, LRT is an isosceles triangle and SLT is a straight line.



Find the value of x.

(c) The diagram below shows a shape, T, and its image, Q, after a transformation.

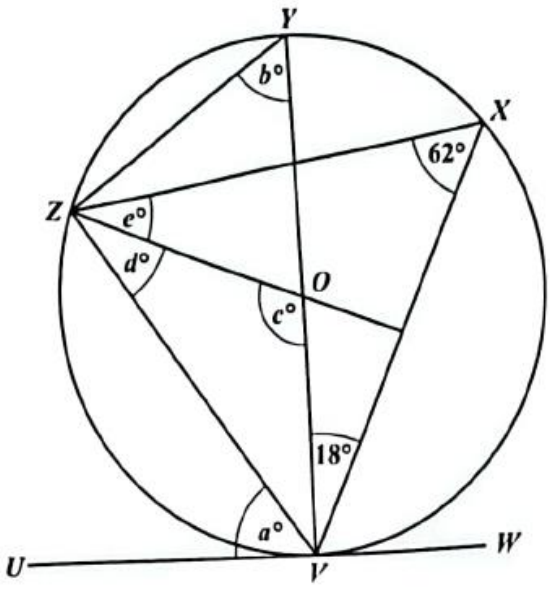


(i) Describe fully the **single** transformation that maps Shape T onto Shape Q.

(ii) On the diagram above, draw the image of Shape T after it undergoes a translation by the vector $\begin{bmatrix} -1 \\ 6 \end{bmatrix}$. Label this image M. **(1 mark)**

May/June 2024 #9

9. (a) V, X, Y and Z lie on the circumference of the circle shown below, centre O, with diameter VY. UW is a tangent to the circle at V. Angle $VXZ = 62°$ and Angle $XVY = 18°$.



(i) State a theorem that justifies the values of EACH of the following angles.

a) Angle $b = 62°$

b) Angle $c = 124°$

c) Angle $OVW = 90°$

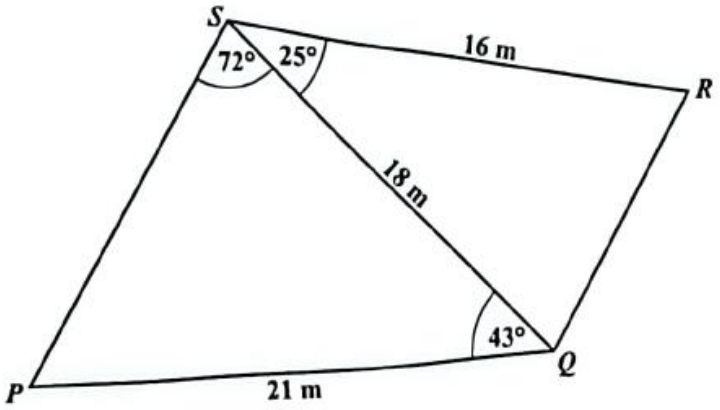
(ii) Find the values of Angles a, d and e. Show ALL working where appropriate.

$\angle a =$

$\angle d =$

$\angle e =$

(b) The diagram below shows a quadrilateral $PQRS$ formed by joining two triangles, PQS and QRS.



(i) Calculate the length of QR.

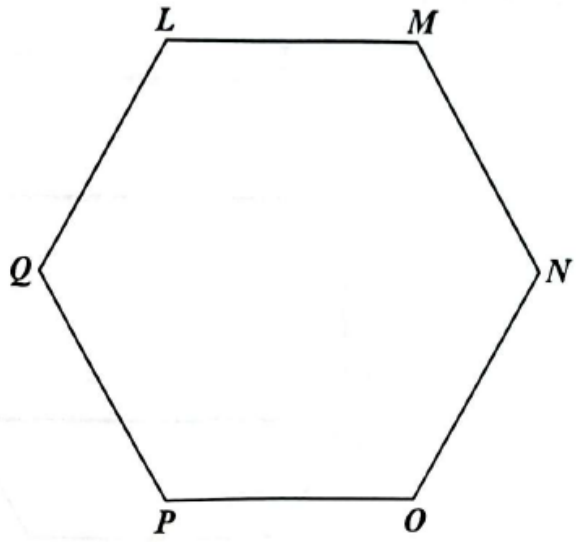
(ii) The bearing of P from S is 205°. Determine the bearing of

a) R from S

b) S from P.

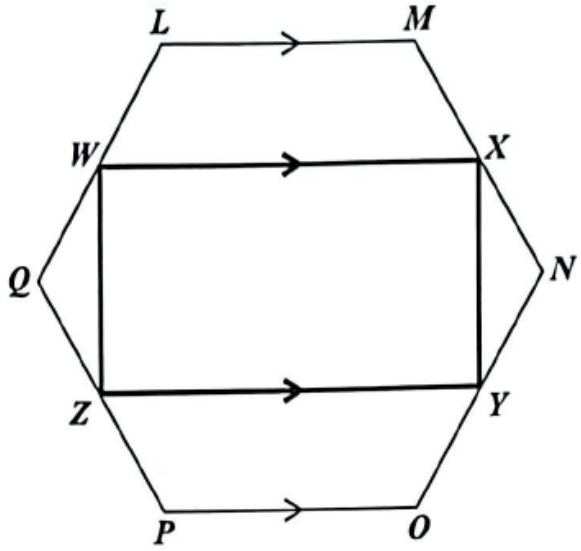
Jan 2025 #3

3. (a) The diagram below shows a **regular** hexagon, $LMNOPQ$, whose side is 8 cm.



(i) Show that the value of Angle PQL is 120°.

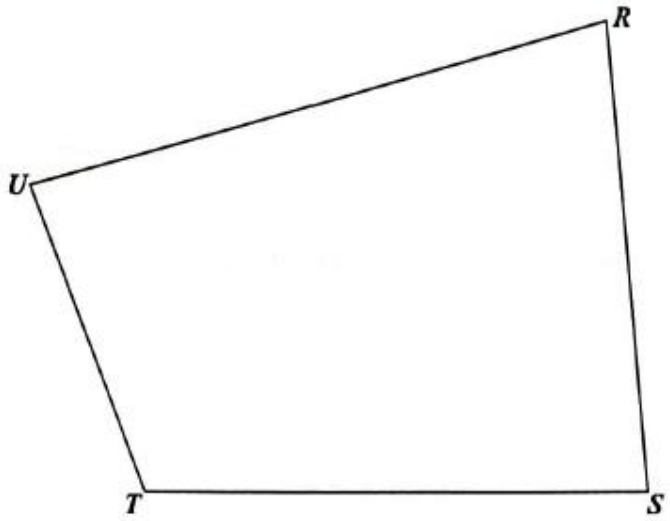
(ii) The vertices of a rectangle, $WXYZ$, touch the sides PQ, QL, MN and NO of the hexagon in Part (a). ZY and WX are parallel to LM and PO.



Calculate the value of Angle LWX.

(b) **In the following parts, show all your construction lines where required.**

The field of a sports club is in the shape of a quadrilateral, *RSTU*. A scaled diagram of this field is shown below.

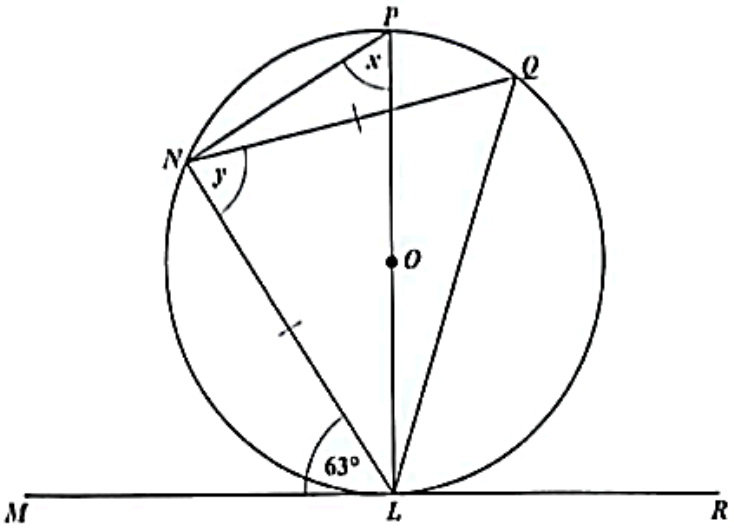


A lamppost is to be erected on the field at a point marked L, so that floodlights can be installed. The point L should be located in such a way that L lies on the perpendicular bisector of the line UR and Angle LST equals 30°.

Using a ruler and compasses only, locate the point L on the field. **(5 marks)**

Jan 2025 #9

9. (a) The diagram below shows a circle with its centre O and the points P, Q, L and N lying on its circumference. $LN = NQ$ and RM is a tangent to the circle at L. Angle $MLN = 63°$.



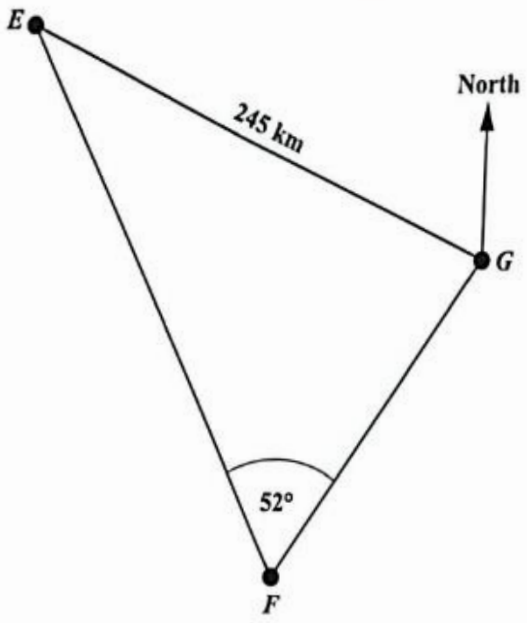
(i) Explain why Angle x and Angle NQL are equal.

(ii) Determine the value of EACH of the following angles. Show detailed working where possible and give a reason for your answer.

a) Angle x

b) Angle y

(b) Two ports, E and G, are on level ground, 245 km apart. The bearing of E from G is 302°. A ship is anchored at F, some distance away from G, on a bearing of 228°. Angle $EFG = 52°$. This information is shown on the diagram below.



(i) a) On the diagram above, insert the angle 228°, the bearing of F from G.

b) Determine the value of Angle FEG.

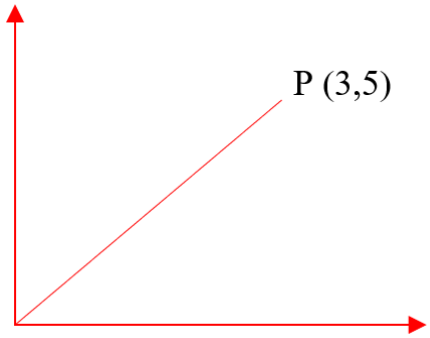
(ii) Calculate GF, the distance the ship is from Port G.

(iii) a) Indicate the point H on the line EF, such that GH is the SHORTEST distance from G to EF.

b) Determine the distance GH.

Section 9 – Vectors and Matrices

Vectors:



Position vector, $\overrightarrow{OP} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$\therefore \overrightarrow{PO} = -\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$

Magnitude of OP, $|OP| = \sqrt{3^2 + 5^2} = \sqrt{36} = 6$

Addition of Vectors:

Example: $\begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+4 \\ -3+1 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

Subtraction of Vectors:

Example: $\begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 2-4 \\ -3-1 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

Multiplication of Vectors:

- By a scalar

  If $\overrightarrow{OP} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

  then $2\overrightarrow{OP} = 2\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

- Two Vectors

  If we have two vectors $\vec{P} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\vec{Q} = \begin{pmatrix} c \\ d \end{pmatrix}$

  then $\vec{P} \cdot \vec{Q} = ad + bc$

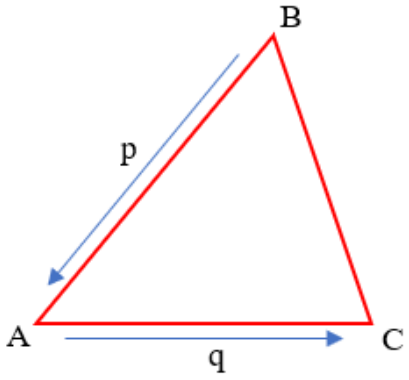
  This is called the **scalar or dot product.**

Displacement Vectors:

If $\overrightarrow{AB}$ = p and $\overrightarrow{AC}$ = q

then $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = -p + q$

This is an alternate route from B to C.

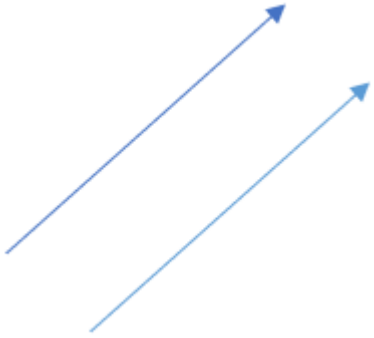


So,

1. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
2. $\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

Parallel Vectors:

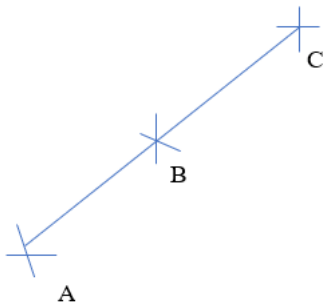
Parallel vectors are multiples of each other. i.e. $a = kb$



Collinear Vectors:

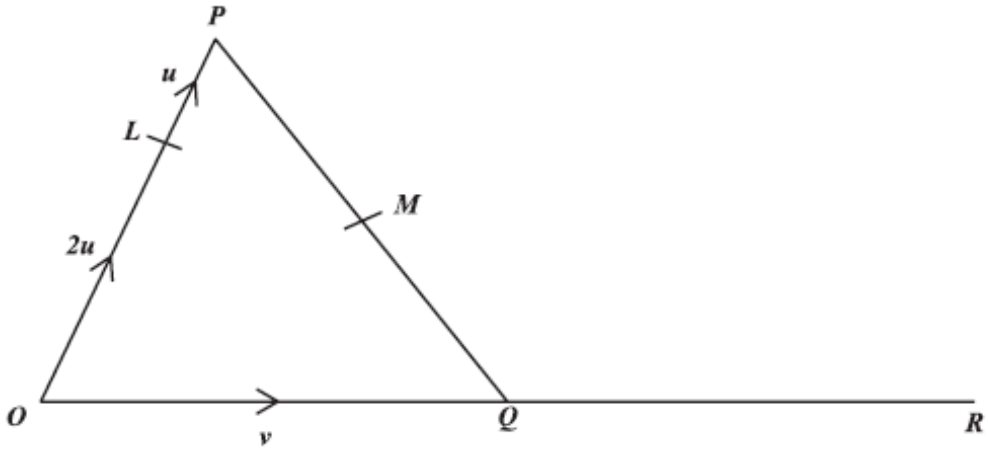
To show that vectors are collinear,

1. Show $\overrightarrow{AB} \parallel \overrightarrow{BC}$
2. Show $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$



Jan 2021 #10

(c) The diagram below shows triangle OPQ in which $\overrightarrow{OP} = 3u$ and $\overrightarrow{OQ} = v$. Q is the midpoint of OR and M is the midpoint of PQ. L is a point on OP such that $OL = \frac{2}{3}OP$.



(i) Write in terms of u and v, an expression for

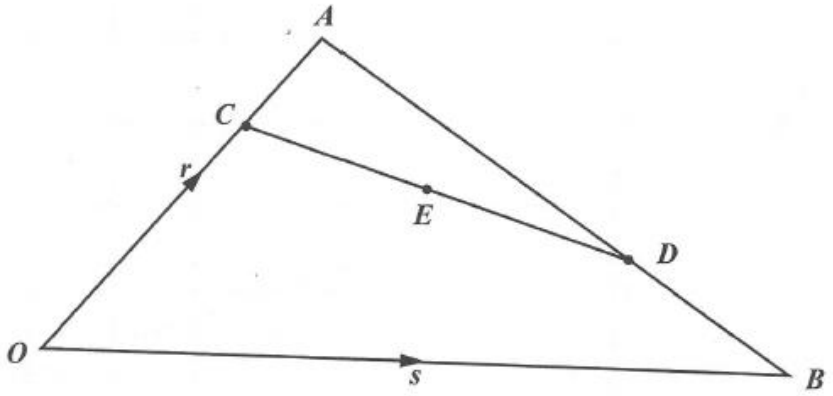
a) $\overrightarrow{LM}$

b) $\overrightarrow{PR}$.

(ii) Prove that the points L, M and R are collinear.

May/June 2021 #10

(c) The diagram below shows triangle OAB in which $\overrightarrow{OA} = r$ and $\overrightarrow{OB} = s$. In addition, E is the midpoint of CD, $OC = \frac{3}{4}OA$ and $AD = \frac{2}{3}AB$.

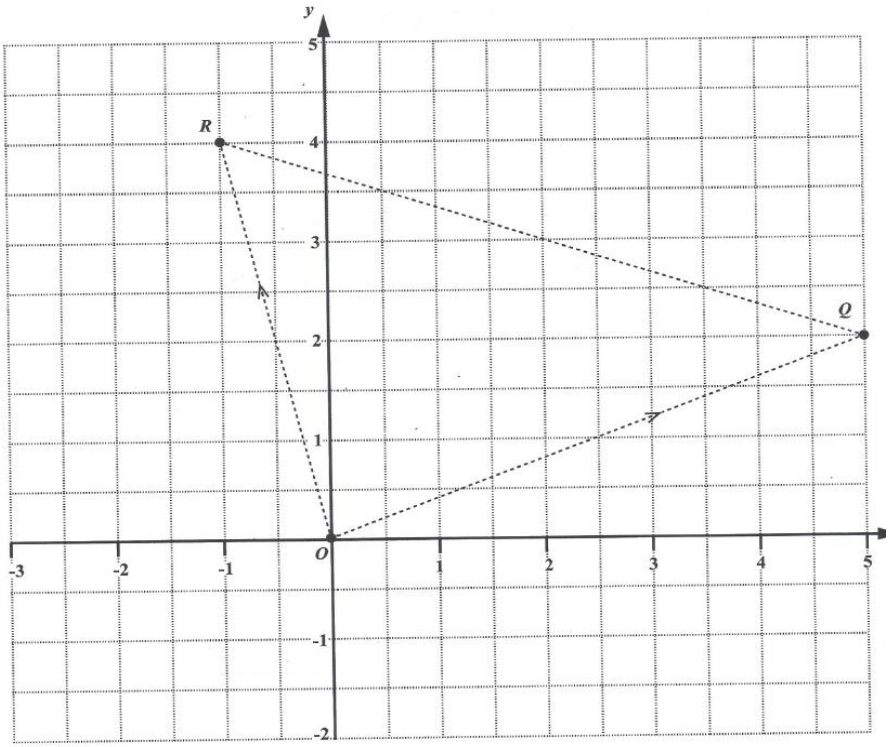


Write in terms of r and s, in the simplest form, an expression for

(i) $\overrightarrow{CD}$

(ii) $\overrightarrow{OE}$.

(d) The points O, Q and R have coordinates (0, 0), (5, 2) and (–1, 4) respectively.

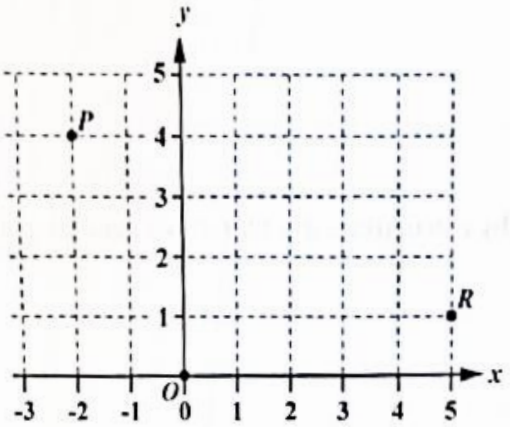


(i) Write $\overrightarrow{OR}$ as a column vector.

(ii) Determine $\left|\overrightarrow{QR}\right|$.

Jan 2022 #10

10. (a) Three points, O, P and R, are shown on the grid below. O is the origin.



(i) Write the position vector of R, $\overrightarrow{OR}$, in the form, $\begin{pmatrix} a \\ b \end{pmatrix}$.

(ii) Another point, Q, is located in such a way that $\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

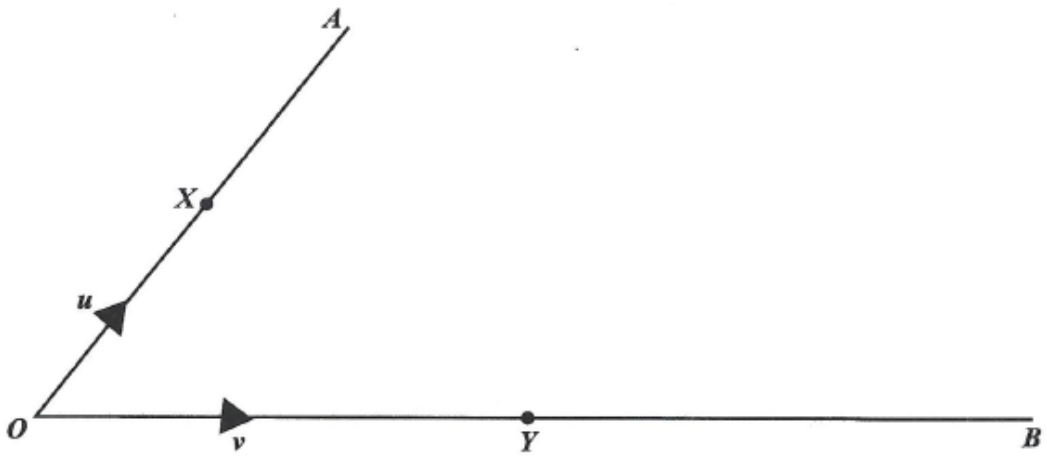
Using this information, plot the point Q on the graph.

(iii) Determine $|\overrightarrow{QR}|$, the magnitude of $\overrightarrow{QR}$.

(iv) Show, **by calculation**, that $OPQR$ is a parallelogram.

May/June 2022 #10

(c) In the diagram below, O is the origin, $\overrightarrow{OX} = u$ and $\overrightarrow{OY} = v$. OX and OY are extended so that X and Y are the midpoints of OA and OB respectively.



(i) Express $\overrightarrow{BX}$ in terms of u and v.

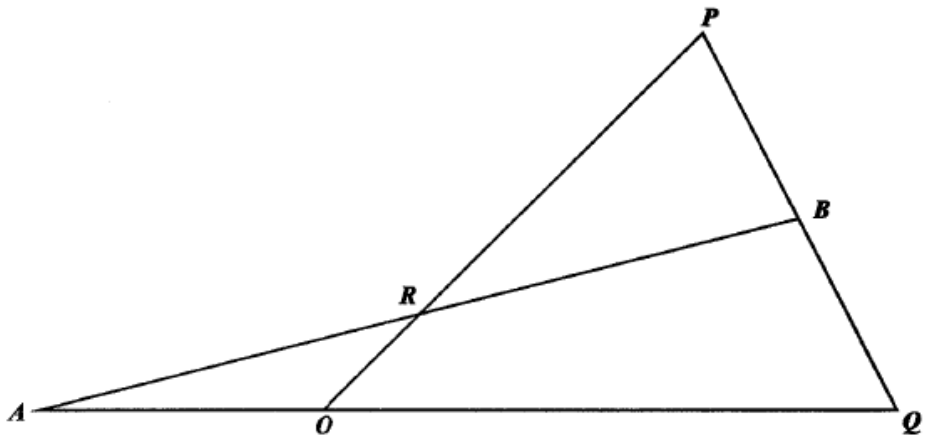
(ii) Given that YA and BX intersect at M and $BM = 2MX$,

a) express $\overrightarrow{BM}$ in terms of u and v.

b) using a vector method, show that the ratio $YM:YA$ is 1:3. Show ALL working.

Jan 2023 #10

(c) In the diagram below, OPQ is a triangle. ARB and AOQ are straight lines.
B is the midpoint of PQ.
R is the midpoint of AB.
$OR : RP = 1 : 3$.
$\overrightarrow{OP} = 4\mathbf{a}$ and $\overrightarrow{OQ} = 8\mathbf{b}$.



Find, in terms of **a** and/or **b**, in its simplest form

(i) $\overrightarrow{PQ}$

(ii) $\overrightarrow{PR}$

(iii) $\overrightarrow{RB}$.

May/June 2023 #10

(c) The following vectors are defined as shown below.

$$\overrightarrow{WX} = \begin{bmatrix} 5 \\ -1 \end{bmatrix} \qquad \overrightarrow{XY} = \begin{bmatrix} -3 \\ 7 \end{bmatrix} \qquad \overrightarrow{ZY} = \begin{bmatrix} 8 \\ -7 \end{bmatrix}$$

Determine EACH of the following.

(i) A vector, other than $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$, that is parallel to $\overrightarrow{WX}$

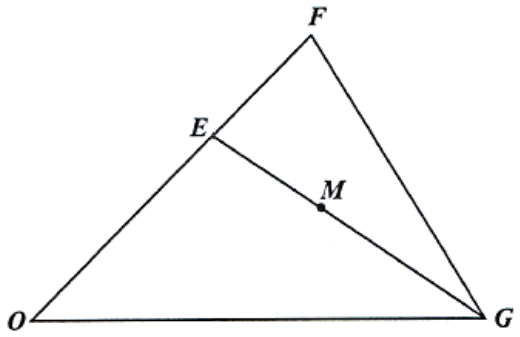
(ii) $\overrightarrow{WY}$

(iii) $\overrightarrow{XZ}$

(iv) $\left|\overrightarrow{XY}\right|$

Jan 2024 #10

10. (a) In the diagram below, O is the origin, $OE = 2EF$ and M is the midpoint of EG. $\overrightarrow{OG} = \mathbf{c}$ and $\overrightarrow{OF} = \mathbf{d}$.



Find in terms of **c** and **d**, in its simplest form,

(i) $\overrightarrow{FG}$

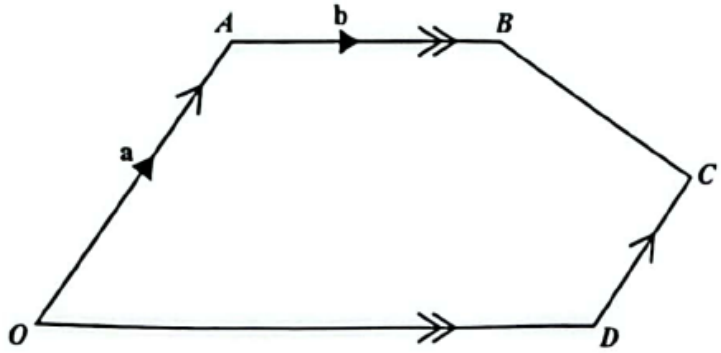
(ii) $\overrightarrow{EG}$

(iii) $\overrightarrow{OM}$.

May/June 2024 #10

(d) In the pentagon $OABCD$, OA is parallel to DC and AB is parallel to OD.

$OD = 2AB$ and $OA = 2DC$. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{AB} = \mathbf{b}$.



Find, in terms of **a** and **b**, in its simplest form,

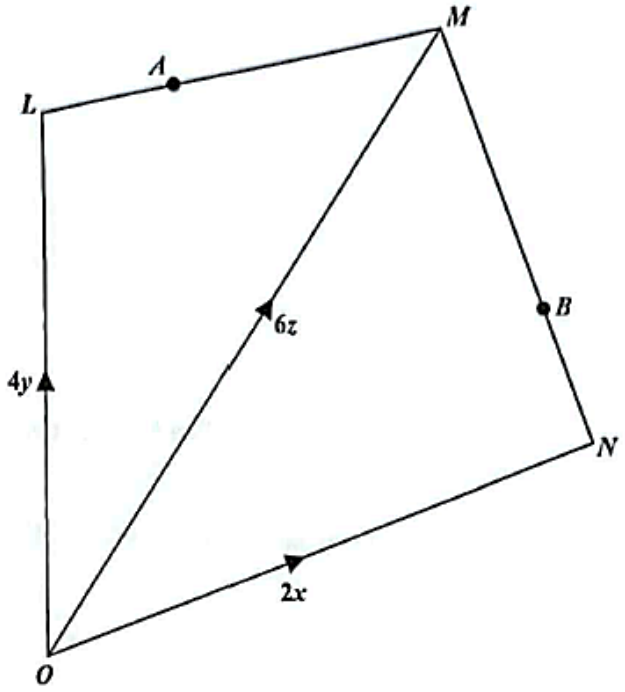
(i) $\overrightarrow{AD}$

(ii) $\overrightarrow{BC}$.

(iii) State the conclusion about $|\overrightarrow{AD}|$ and $|\overrightarrow{BC}|$ that can be drawn from your responses in (i) and (ii).

Jan 2025 #10

10. (a) The diagram below shows quadrilateral $OLMN$, in which O is the origin, $\overrightarrow{OL} = 4\boldsymbol{y}$, $\overrightarrow{OM} = 6\boldsymbol{z}$ and $\overrightarrow{ON} = 2\boldsymbol{x}$. The point A lies on LM such that $LA : AM = 1:2$ and the point B on MN such that $MB : BN = 2:1$.



(i) Express, in its simplest form, $\overrightarrow{MN}$ in terms of x and z.

(ii) a) Find, in terms of x and y, in its simplest form, an expression for $\overrightarrow{LN}$.

b) Show that $\overrightarrow{AB}$ equals $\frac{2}{3}(2\boldsymbol{x} - 4\boldsymbol{y})$.

(iii) Based on your results in Part (ii), state TWO geometric properties relating LN to AB.

Matrices:

A matrix is a set of elements in rows or columns.

Example: $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{matrix} \rightarrow row\ 1 \\ \rightarrow row\ 2 \end{matrix}$

$\begin{matrix} \downarrow & \downarrow \\ Column\ 1 & Column\ 2 \end{matrix}$

Size of matrix = number of rows × number of columns

Example:
State the sizes of the following matrices :

1. $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 3 \times 1 \rightarrow$ Column Matrix

2. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 2 \times 2 \rightarrow$ Square Matrix (N.B. This is an Identity Matrix)

3. $(4 \quad 2) = 1 \times 2 \rightarrow$ Row Matrix

4. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 2 \times 2 \rightarrow$ Null Matrix

Addition and Subtraction of Matrices:

If we have:

$$A = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \ , \qquad B = (3 \quad -1) \ , \ C = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

To add or subtract matrices:

1. Matrices must be the same size.
2. Add or subtract the element in the same position

Example 1:

$$A + B = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + (3 \quad -1)$$

$2 \times 1 \qquad 1 \times 2$

Since matrix A and matrix B are not the same size, we cannot add them.

Example 2:

$$A + C = \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}1+4\\2+3\end{pmatrix} = \begin{pmatrix}5\\5\end{pmatrix}$$

$2 \times 1 \qquad 2 \times 1$

Multiplication of Matrices:

a) By a scalar

If we have $A = \begin{pmatrix}a & b\\c & d\end{pmatrix}$

then $kA = k\begin{pmatrix}a & b\\c & d\end{pmatrix}$

$$= \begin{pmatrix}ka & kb\\kc & kd\end{pmatrix}$$

Example: If $A = \begin{pmatrix}2 & -3\\4 & 5\end{pmatrix}$ then

1. $2A = 2\begin{pmatrix}2 & -3\\4 & 5\end{pmatrix} = \begin{pmatrix}4 & -6\\8 & 10\end{pmatrix}$

2. $3A = 3\begin{pmatrix}2 & -3\\4 & 5\end{pmatrix} = \begin{pmatrix}6 & -9\\12 & 15\end{pmatrix}$

b) Two Matrices

To multiply two matrices:

1. No. of columns in the first matrix must be equal to the no. of rows in the second matrix. i.e. $a \times \boxed{b \qquad b} \times c$

2. The resulting matrix size is the no. of rows in matrix one by the no. of rows in matrix two. i.e. Resulting Matrix = $a \times c$

3. Always multiply the rows in matrix one by the columns in matrix two.

Examples:

If $A = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 3 & -1 \end{pmatrix}$

2×2 $\qquad$ 2×2

1. $AB = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 3 & -1 \end{pmatrix}$

$$= \begin{pmatrix} (2 \times 10) + (-1)(3) & (2)(1) + (-1)(-1) \\ (3 \times 0) + (2 \times 3) & (3)(1) + (2)(-1) \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 3 \\ 6 & 1 \end{pmatrix} \rightarrow 2 \times 2 \text{ matrix}$$

2. $BA = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 3 & -1 \end{pmatrix}$

$$= \begin{pmatrix} (0 \times 2) + (1 \times 3) & (0 \times -1) + (1 \times 2) \\ (3 \times 2) + (-1 \times 3) & (3 \times -1) + (2 \times -1) \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ 3 & -5 \end{pmatrix}$$

Determinant of a Matrix:

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then its determinant is denoted by det A *or* $|A|$.

$|A| = ad - bc$

Examples:

1. $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$

$|A| = (2)(4) - (-1)(3)$

$= 8 + 3 = 11$

2. $A = \begin{pmatrix} 2 & 4 \\ -1 & -3 \end{pmatrix}$

$|A| = (2)(-3) - (4)(-1)$

$= -6 - (-4)$

$= -6 + 4 = -2$

3. $B = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

$|A| = (1)(4) - (2)(2)$

$= 4 - 4 = 0$

If the det. is $\neq 0$ then the matrix is said to be non-singular.

If the det. $= 0$ then the matrix is said to be singular.

Examples:

1. If $A = \begin{pmatrix} 2 & p \\ 3 & 1 \end{pmatrix}$ is a singular matrix, Find the value of p.

Since matrix is singular, $|A| = 0$

$\therefore |A| = (2)(1) - (p)(3) = 0$

$2 - 3p = 0$

$-3p = -2$

$p = \frac{-2}{-3}$

$p = \frac{2}{3}$

2. If $A = \begin{pmatrix} 2 & 3 \\ p & 4 \end{pmatrix}$ is a singular matrix, Find the value of p.

Since matrix is singular, $|A| = 0$

$\therefore |A| = (2)(4) - (3)(p) = 0$

$8 - 3p = 0$

$-3p = -8$

$p = \frac{-8}{-3}$

$p = \frac{8}{3}$

Inverse of a Matrix:

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then its inverse is denoted by A^{-1}.

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

N.B. To find the inverse of the matrix A, the matrix must be non-singular.

Examples:

1. If $A = \begin{pmatrix} 2 & -3 \\ 1 & 3 \end{pmatrix}$

$$|A| = (2)(3) - (-3)(1)$$
$$= 6 + 3 = 9$$
$$\therefore A^{-1} = \frac{1}{9}\begin{pmatrix} 3 & 3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3/9 & 3/9 \\ -1/9 & 2/9 \end{pmatrix}$$

2. If $A = \begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix}$

$$|A| = (2)(-3) - (-4)(1)$$
$$= -6 + 4 = -2$$
$$\therefore A^{-1} = \frac{1}{-2}\begin{pmatrix} -3 & 4 \\ -1 & 2 \end{pmatrix}$$

3. If $B = \begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$

$$|A| = (2)(3) - (6)(1)$$
$$= 6 - 6 = 0$$

$\therefore B$ is singular

$\therefore$ No inverse

$$\frac{1}{0} = \infty$$

Solving Simultaneous Equations using the Matrix Method:

When given 2 equations, we can solve simultaneously using the following steps:

1. Write the equation in matrix form. i.e. $Ax = b$
   Where A = the coefficients
   x = the unknown variables
   b = the constants

2. Find A^{-1}

3. $x = A^{-1} \cdot b$

Example:

$2x + y = 3$

$3x - 2y = 1$

- Write in Matrix form

$$\underset{A}{\begin{pmatrix}2 & 1\\ 3 & -2\end{pmatrix}} \underset{x}{\begin{pmatrix}x\\ y\end{pmatrix}} = \underset{b}{\begin{pmatrix}3\\ 1\end{pmatrix}}$$

- Find A^{-1}

$$|A| = (2)(-2) - (1)(3)$$
$$= -4 - 3 = -7$$

$$\therefore A^{-1} = \frac{1}{-7}\begin{pmatrix}-2 & -1\\ -3 & 2\end{pmatrix}$$

- $x = A^{-1} \cdot b$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \frac{1}{-7}\begin{pmatrix}-2 & -1\\ -3 & 2\end{pmatrix}\begin{pmatrix}3\\ 1\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \frac{1}{-7}\begin{pmatrix}-6 + (-1)\\ -9 + 2\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \frac{1}{-7}\begin{pmatrix}-7\\ -7\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-7/-7\\ -7/-7\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}1\\ 1\end{pmatrix}$$

Transformation Matrices:

Thumb Index Finger	Transformation	Left Hand Rule
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Identity Matrix	Stays the same	
PALM FACING YOU A REFLECTION		
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Reflection in the line y=x	
$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	Reflection in the line y=-x	
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Reflection in the x axis	
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	Reflection in the y axis	
BACK OF HAND FACING YOU A ROTATION		
$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	Rotation 90° clockwise	
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	Rotation 180°	
$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	Rotation 90° anticlockwise	

Jan 2021 #10

10. (a) Given the matrix $W = \begin{pmatrix} 3 & 6 \\ -2 & 5 \end{pmatrix}$, determine

(i) the 2×2 matrix, L, such that $W + L = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(ii) the 2×2 matrix, P, such that $WP = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(b) A right-angled triangle, M, has vertices $X(1, 1)$, $Y(3, 1)$ and $Z(3, 4)$. When M is transformed by the matrix $N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the image is M'.

Find the coordinates of the vertices of M'.

May/June 2021 #10

10. (a) (i) Calculate the matrix product $\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix}$.

(ii) State why the two matrices in (a) (i) are conformable for multiplication.

(b) Determine the inverse of $\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$.

Jan 2022 #10

(b) Calculate the value of x and the value of y in the matrix equation below.

$$\begin{pmatrix} 1 & 5 \\ 2 & y \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 2 & 9 \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$$

(c) A transformation, T, represented by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, maps $S(2, 5)$ onto $S'(5, 2)$.

Describe fully the single transformation T.

May/June 2022 #10

10. (a) The transformation matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents a rotation of 90° anticlockwise about the origin O.

The transformation matrix $\boldsymbol{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the straight line with equation $y = -x$.

(i) Write the coordinates of P', the image of the point $P(7, 11)$ after it undergoes a rotation by 90° anticlockwise about the origin, O.

(ii) T is the combined transformation of A followed by B. Determine the elements of the matrix representing the transformation T.

(iii) Describe, geometrically, the transformation represented by T.

(b) The 2 × 2 matrix C is defined, in terms of a scalar constant k, by

$$C = \begin{pmatrix} 3 & k \\ 6 & 4 \end{pmatrix}.$$

Determine the value of k, given that the matrix C is singular.

Jan 2023 #10

10. (a) Three matrices Q, R and S are as follows:

$$Q = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 6 \\ -5 & 4 \end{pmatrix}, \quad S = \begin{pmatrix} 2 & 7 \\ 4 & -1 \\ -8 & 9 \end{pmatrix}.$$

(i) Explain why the matrix product QS is NOT possible.

(ii) State the order of the matrix product SR.

(iii) Calculate the matrix product QR.

(b) Given that $A = \begin{pmatrix} 4 & -1 \\ -7 & x \end{pmatrix}$, determine the value of x when $|A| = 5$.

May/June 2023 #10

10. (a) The matrices A and B represent the transformations given below.

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents an anticlockwise rotation of 90° about the origin, O.

$B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the straight line $y = -x$.

(i) Determine the elements of the matrix C which represents an anticlockwise rotation of 90° about the origin, O, followed by a reflection in the straight line $y = -x$.

(ii) Describe, geometrically, the single transformation represented by C.

(b) A transformation, T, is defined by the following 2×2 matrix.

$$T = \begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

T maps the point (2, 3) onto the point (8, 15).

Determine the value of k.

Jan 2024 #10

(b) The matrices P, Q and R are given below, in terms of the scalar constants a, b and c, as

$$P = \begin{pmatrix} 3 & -9 \\ a & 7 \end{pmatrix}, \quad Q = \begin{pmatrix} -1 & b \\ -4 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} c & -3 \\ -4 & 8 \end{pmatrix}.$$

Given that $P + Q = R$, find the value of a, b and c.

(c) Solve the following pair of simultaneous equations using a matrix method.

$$5x - 2y = 44$$
$$2x + 3y = 10$$

May/June 2024 #10

10. (a) The determinant of the matrix $\begin{pmatrix} 6 & 2v \\ -5 & -v \end{pmatrix}$ is 24.

Calculate the value of v.

(b) The matrices L and M are defined as follows.

$$L = \begin{pmatrix} 9 & 5 \\ 3 & 2 \end{pmatrix}, \quad M = \begin{pmatrix} 2 \\ -4 \end{pmatrix}.$$

Evaluate EACH of the following.

(i) The matrix product LM

(ii) L^{-1}, the inverse of L

(c) $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$.

If P is the point $(-2, 3)$, determine the coordinates of Q.

Jan 2025 #10

(b) Determine the values of the unknowns in EACH of the matrix equations below.

(i) $\begin{pmatrix} 4 & 0 \\ -2 & 5 \end{pmatrix} + \begin{pmatrix} x & 2 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$

(ii) $\begin{pmatrix} 5 & -3 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} a & 2 \\ c & -1 \end{pmatrix} = \begin{pmatrix} -10 & 13 \\ 17 & 1 \end{pmatrix}$