2023

1. (a) (i) Let p and q be any two propositions. Complete the truth table below.

| p | q | p ⟶ q | q ⟶ p | (p ⟶ q) ∧ (q ⟶ p) |
|---|---|---|---|---|
| T | T | | | |
| T | F | | | |
| F | T | | | |
| F | F | | | |

**[3 marks]**

(ii) Hence, state whether the statements q ⟶ p and (p ⟶ q) ∧ (q ⟶ p) are logically equivalent. **Justify your response.**

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**[2 marks]**

(b) Let $x$ and $y$ be negative real numbers and let $z$ be any real number. Use a counter example to show that the statement "if $x > y$ then $xz > yz$" is false.

(c) The expression $f(x) = 6x^3 + px^2 + qx + 2$ is divisible by $2x - 1$ and has a remainder of 2 when divided by $x - 1$.

Calculate the values of $p$ and $q$.

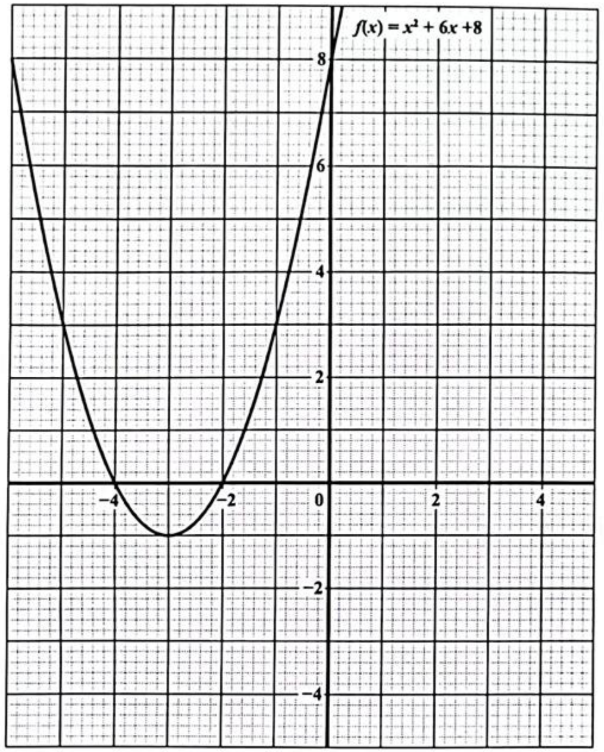
(d) (i) Solve the logarithmic equation $\log_3(x^2 - 9) - \log_3(x + 3) = 3$.

(ii) Show that $\sqrt{320x^3} + \sqrt{125x^3}$ simplifies to $13x\sqrt{5x}$ .

2. (a) Let $f(x) = 7x + 2$. Prove that $f$ is bijective.

(b) The roots of the cubic equation $3x^3 - x^2 - 2x + 1 = 0$ are $\alpha$, $\beta$ and $\gamma$. Determine the equation whose roots are $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$.

(c) The diagram below shows the graph of the curve $f(x) = x^2 + 6x + 8$.




(i) On the axes provided above, sketch and label the graph of $g(x) = |x^2 + 6x + 8|$. **[3 marks]**

(ii) On the same axes, sketch and label the inverse of $f$ for $x \geq -3$. **[5 marks]**

(d) Given that $g(x) = \dfrac{2x + 3}{x + 3}$, prove that $g^{-1}(2)$ does not exist.

2022

(a) (i) Let **p** and **q** be any two propositions. Complete the truth table below.

| p | q | ~q | (~q ∧ p) | p ∨ (~q ∧ p) |
|---|---|---|---|---|
| T | T | | | |
| T | F | | | |
| F | T | | | |
| F | F | | | |

**[3 marks]**

(ii) Hence, state whether the statements (~q ∧ p) and p ∨ (~q ∧ p) are logically equivalent. **Justify your response.**

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**[2 marks]**

(b) Determine the values of $a$, $b$, and $c$, such that

$$2x^2 - 7x + 12 = a(x-2)(x-1) - b(x-3) + c.$$

(c) Solve the inequality $|3x+2| \geq 4$.

(d) Solve the logarithmic equation $\log_5(x+2) + \log_5(x+6) = 1$.

(a) Let $f(x) = \dfrac{3x+1}{x}$ and $g(x) = e^{-2x} + 1$.

(i) Show that $f^{-1}(x) = \dfrac{1}{x-3}$.

(ii) Hence, or otherwise, write an expression for $(f^{-1} \circ g)(x)$.

(b) Solve the equation $6 - \dfrac{7}{2^{2x}} - \dfrac{3}{4^{2x}} = 0$.

(c) The function $f(x) = 2x^3 - px^2 + qx - 10$ is divisible by $x - 1$ and has a remainder of $-6$ when divided by $x + 1$.

Calculate the values of $p$ and $q$.

(d) The roots of the equation $2x^3 - x^2 + 3x - 1 = 0$ are $\alpha$, $\beta$, and $\gamma$.

Given that $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{3}{2}$ and $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$, determine the equation whose roots are $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$.

2021

1. (a) (i) Let **p** and **q** be any two propositions. Complete the truth table below.

| p | q | ~p | ~q | p ∨ q | ~(p ∨ q) | ~p ∨ ~q |
|---|---|---|---|---|---|---|
| T | T | | | | | |
| T | F | | | | | |
| F | T | | | | | |
| F | F | | | | | |

**[4 marks]**

(ii) Hence, state whether the statements ~(p ∨ q) and ~p ∨ ~q are logically equivalent. **Justify your response.**

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**[2 marks]**

(b) (i) Write the converse of the statement "$n$ **is an integer** $\Rightarrow$ $n^2$ **is an integer**".

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**[1 mark]**

(ii) For two real numbers $x$ and $y$, the operation * is given by $x * y = x^2 + y^2$. Prove that * is closed in **R**.

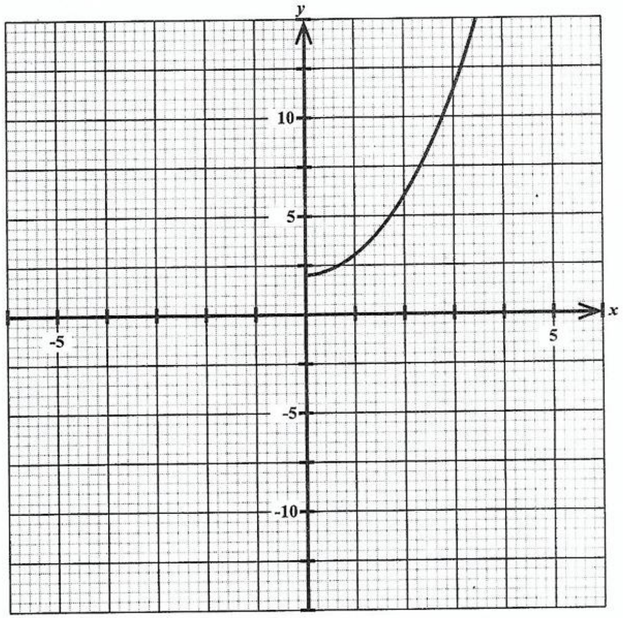
(iii) Determine whether the operation * is commutative.

(c) The function $f(x) = ax^3 + 3x^2 - b$ is divisible by $2x - 1$ and has a remainder of $-5$ when divided by $x + 2$.

Calculate the values of $a$ and $b$.

(d) Solve the logarithmic equation $\log_2 x + \log_4 x + \log_{16} x = 7$.

2. (a) The diagram below shows the graph of $f(x) = x^2 + 2$ for $x \geq 0$.




(i) On the graph

a) Sketch the inverse of $f$

b) Show that the inverse of $f$ is a function. **[4 marks]**

(ii) Prove that $f$ is one to one.

(iii) Determine whether $f$ is onto.

(b) Solve the equation $|x^2 - 4| = 3x - 2$.

(c) Given that α, β, and γ are the roots of the equation $2x^3 - x^2 + 3x - 1 = 0$, determine the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$.

2019

1. (a) The quadratic expression $f(x) = ax^2 + 12x + b$ is divisible by $x - 3$ and has a remainder $-27$ when divided by $x + 6$.

(i) Show that the values of the constants $a$ and $b$ are 3 and −63, respectively.

(ii) Hence, determine the factors of $f$.

(b) Solve, for real values of $x$, the inequality $|3x - 4| \leq 6$.

(c) A binary operation is defined on the set of rational numbers by $a * b = \frac{ab}{2}$. Prove that $*$ is commutative.

(d) Use mathematical induction to prove that $5^n - 1$ is divisible by 4 for $n \in \mathbf{N}$.

(a) Let $A = \{x : x \in \mathbf{R}, x \geq 1\}$.

A function $f : A \rightarrow \mathbf{R}$ is defined as $f(x) = x^2 - x$. Show that $f$ is one-to-one.

Let $f(x) = 3x + 1$ and $g(x) = e^{3x}$.

(i) Determine $f^{-1}$.

(ii) Determine $f^{-1} \circ g$.

(c) Solve the equation $3 - \frac{4}{9^x} - \frac{4}{81^x} = 0$.

(d) Let the roots of the equation $2x^3 - 5x^2 + 4x + 6 = 0$ be $\alpha, \beta$ and $\gamma$.

(i) State the values of $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$ and $\alpha\beta\gamma$.

(ii) Hence, or otherwise, determine the equation with roots $\alpha^2, \beta^2$ and $\gamma^2$.

**Note:** $(\alpha\beta)^2 + (\alpha\gamma)^2 + (\beta\gamma)^2 = (\alpha\beta + \alpha\gamma + \beta\gamma)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

2024

(a) Given that $f(x) = \frac{4}{x-5}$ and $g(x) = \frac{1}{x}$, determine the domain of the composite function $f(g(x))$.

(b) Solve the inequality $-3|2x - 5| + 2 \geq -4$.

(c) Solve the logarithmic equation $\log_5 x - 4 \log_x 5 = -3$.

(d) Prove that the function $f(x) = 3x - 2$ is bijective.

(a) The roots of the cubic equation $x^3 - 9 = 0$ are $\alpha$, $\beta$ and $\gamma$. Determine the with roots $(\alpha - 1)$, $(\beta - 1)$ and $(\gamma - 1)$, given that
$(\alpha - 1)(\beta - 1) + (\alpha - 1)(\gamma - 1) + (\beta - 1)(\gamma - 1) = 12$.

(b) The function $f(x) = 2x^3 - px^2 + qx - 5$ is divisible by $2x - 1$ and has a remainder of 7 w divided by $x - 1$. Calculate the values of $p$ and $q$.

(c) Let $P$ be the proposition that if $n^3 + 3$ is odd then $n$ is even for all integers $n$. Provide a proof by contradiction for this proposition.

Hint: Let $n^3 + 3$ be odd and assume that $n$ is odd. Show that the assumption that $n$ is odd is incorrect.

Logic and Reasoning

If we have the conditional statement, $\sim\mathbf{p} \rightarrow \mathbf{q}$ then:

- Converse:
- Inverse:
- Contrapositive:

Prove $(\mathbf{p} \vee \mathbf{q}) \wedge \sim\mathbf{p} \equiv \sim\mathbf{p} \wedge \mathbf{q}$.

Prove $\sim(\mathbf{p} \wedge \sim\mathbf{q}) \equiv \sim\mathbf{p} \vee \mathbf{q}$.

Prove $\sim(\mathbf{p} \wedge \sim\mathbf{q}) \wedge (\sim\mathbf{p} \vee \sim\mathbf{q}) \equiv \sim\mathbf{p}$.

PMI

Using P.M.I. show that $P_n: 6^n - 1$ is divisible by 5, $\forall\, n \in \mathbb{N}$.

Using P.M.I. show that $3^{4n} - 1$ is divisible by 16, $\forall\, n \geq 1$.

Using P.M.I. show that $\sum_{r=1}^{n} r = \frac{1}{2}(n)(n+1)$, $\forall\, n \geq 1$.

Using P.M.I. show that $\sum_{r=1}^{n} (4r - 3) = n(2n - 1)$, $\forall\, n \in \mathbb{N}$.

Using P.M.I. show that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$, $\forall\, n \in \mathbb{N}$.

Past Paper Question – 2017 Paper 2

Use mathematical induction to prove that,

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n.(n+1)} = \frac{n}{n+1} \quad \text{for } n \in \mathbb{N}.$$

Surds

Simplify $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Simplify

$$\frac{3\sqrt{5}+2\sqrt{3}}{2\sqrt{5}-4\sqrt{3}}$$

Past Paper Question – 2008 Paper 2 ROR #1 b) (ii)

Without using calculators or tables, evaluate

$$\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} + \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

Past Paper Question – 2005 Paper 2 #3 a)

Show

$$\sqrt{11}+\sqrt{7} = \frac{4}{\sqrt{11}-\sqrt{7}}$$

Past Paper Question – 2004 Paper 2 #3

(i) Show

$$\frac{4}{2^x} = 2^{2-x}$$

(ii) Solve $2^x + 2^{2-x} = 5$

Past Paper Question – 2008 Paper 2 TT #2 a)

Without using calculators or tables, evaluate,

$$\sqrt{\frac{27^{10}+9^{10}}{27^4+9^{11}}}$$

Logs

Past Paper Question – 2009 Paper 2 #1 c)

Solve the equation $\log_4 x = 1 + \log_2 2x \ , x > 0$.

Past Paper Question – 2012 Paper 2 #2 c)

Without the use of calculators or tables, evaluate

(i) $\log_{10}\left(\frac{1}{3}\right) + \log_{10}\left(\frac{3}{5}\right) + \log_{10}\left(\frac{5}{7}\right) + \log_{10}\left(\frac{7}{9}\right) + \log_{10}\left(\frac{9}{10}\right)$.

(ii) $\sum_{r=1}^{99} \log_{10}\left(\frac{r}{r+1}\right)$.

(ii) $\sum_{r=1}^{99} \log_{10}\left(\frac{r}{r+1}\right)$.

Past Paper Question – 2014 Paper 2 #2 b)

Given that $a^3 + b^3 + 3a^2b = 5ab^2$, show that $3 \log\left(\frac{a+b}{2}\right) = \log a + 2 \log b$.